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THE

PHYSICAL REVIEW.

SOME APPLICATIONS OF THE ELECTROMAGNETIC EQUATIONS TO MATTER, SUPPOSED TO BE CONSTITUTED OF POSITIVE AND NEGATIVE ELECTRONS.

By Albert C. Crehore.

THERE is good reason to believe that all so-called gross matter in its neutral state and in the passive condition, which constitutes the very large majority of the matter making up the mass of the earth, is composed of atoms in a practically stable condition, which endure for long periods of time, and that each of these atoms is made of positive and negative electrons, the negative revolving in approximately circular orbits about a center determined by the positive electrons.

The initiative in the following investigations is the belief that all the phenomena of nature will ultimately find their complete and adequate explanation in terms of simpler or more fundamental conceptions by means of an application of the proper theory of the electromagnetic field to the individual electrons making up the atoms. The theorist demands first of all the simplest possible conditions subject to the smallest number of variations, and hence the steady states of these atoms and the matter composed of them are obviously the first to be treated.

There is a tendency among scientists at the present time to take the view that electromagnetic theory is insufficient to account for the atomic phenomena which have been observed, particularly those connected with radiation. This temporary laying aside of the so-called classical dynamics has certainly resulted in much that is good. It has, as one instance, pointed out the existence of a new constant, Planck's "h," which seems to be universally connected with atomic phenomena. But, let us hope that the laying aside of the electrodynamics is only temporary. The failures of the theory really form the pivotal point for future progress, for it is at such points only that we may discover at what point the theory departed from the facts. There are known to be other possible

forms of the potential function, for example, which would be quite as much in harmony with electromagnetic theory as the one on which the most work has been done up to the present time. It is by a closer examination of those cases which fail that we may look for an inspiration to point out the way to more correct ideas. If one believes in the reality of an aether and regards the true electromagnetic equations merely as the expression of its properties in formal terms, it becomes very difficult to conceive that the individual electrons can escape from its domain, and increases the belief that they must be the very first to come under its sway.

Although the derivation of the general equation for the instantaneous mechanical force with which one moving charge affects another is given in the modern textbooks that deal with this subject, yet, since it is our purpose to trace back to their source the origin of any particular term in the final expression for the mechanical force, we will give the derivation in more detail than would otherwise be desirable.

If we desire first of all to know something of properties of matter as a whole, or in the gross, its rigidity, and tendency to maintain a fixed volume under great changes of pressure, and the possibility of forming crystalline structures, the problem resolves itself in its ultimate analysis into that of finding the average mechanical force which one single negative electron exerts upon a second electron, each revolving at uniform angular velocities in circular orbits. The actual forces between the atoms are merely the sums of the various component parts making up the atom.

The fundamental conceptions of an electromagnetic theory are expressed in terms of the electric and magnetic forces by the following equations. $\mathbf{E}$ and $\mathbf{H}$ are the electric and magnetic forces respectively.

$$c \operatorname{curl} \mathbf{E} = -\dot{\mathbf{H}} \tag{1}$$

and

$$\operatorname{div.} \mathbf{H} = 0. \tag{2}$$

In view of (2) it is possible to write

$$\mathbf{H} = \operatorname{curl} \alpha, \tag{3}$$

and α may be defined as the vector potential. Substituting this in (1) we get

$$\operatorname{curl} \left(\mathbf{E} - \frac{1}{c} \dot{\alpha} \right) = 0, \tag{4}$$

whence

$$\mathbf{E} = -\nabla\phi - \frac{1}{c} \dot{\alpha}. \tag{5}$$

This latter equation may be regarded as defining the scalar potential ϕ.

It is recognized that α is not completely determined by (3), and that a large number of different forms for it are possible. The most of the work in the modern theory has been based upon the following condition imposed upon the potentials ϕ and α,

$$\text{div. } \alpha + \frac{1}{c}\dot{\phi} = 0. \tag{6}$$

To obtain the values of the potentials from this use is made of the fundamental equations of definition

$$\text{div. } \mathbf{E} = 4\pi\rho \tag{7}$$

and

$$c \text{ curl } \mathbf{H} = \dot{\mathbf{E}} + 4\pi\rho\mathbf{q}, \tag{8}$$

whence the assumption concerning the potentials (6) leads to the two equations

$$\nabla^2\phi - \frac{1}{c^2}\ddot{\phi} = -4\pi\rho \tag{9}$$

and

$$\nabla^2\alpha - \frac{1}{c^2}\ddot{\alpha} = -4\pi\rho\frac{\mathbf{q}}{c}. \tag{10}$$

Integrals of these two equations have been obtained by Lorentz, namely

$$\phi = \int [\rho] d\Omega/R \tag{11}$$

$$\alpha = \int \left[\rho\frac{\mathbf{q}}{c}\right] d\Omega/R, \tag{12}$$

in which the square brackets indicate that the quantities within them are to be taken at a time $t - (R/c)$ and not at the actual time t when we want the potentials. When the charges are to be considered as point charges it has been shown that the integration of these latter equations leads to the simple expressions for the potentials

$$\phi = \frac{e_2}{\left[R\left(1 - \frac{\mathbf{q}_2\cdot\mathbf{R}}{cR}\right)\right]}, \tag{13}$$

$$\alpha = \frac{e_2\mathbf{q}_2}{c\left[R\left(1 - \frac{\mathbf{q}_2\cdot\mathbf{R}}{cR}\right)\right]}. \tag{14}$$

An additional fundamental equation due to Larmor and Lorentz is required, to derive the mechanical force exerted by one charge upon the other*

* The vector notation of Gibbs is employed throughout.

$$\mathbf{F} = e_1\left(\mathbf{E} + \frac{1}{c}\,\mathbf{q}_1 \times \mathbf{H}\right), \tag{15}$$

which gives from (3) and (5)

$$\mathbf{F} = e_1\left(-\nabla\phi - \frac{1}{c}\frac{\partial\alpha}{\partial t} + \frac{1}{c}\,\mathbf{q}_1 \times \mathbf{H}\right), \quad \text{where} \quad \mathbf{H} = \nabla \times \alpha. \tag{16}$$

Performing the operations upon the potentials for point charges (13) and (14) indicated, we obtain the final values of the mechanical force in terms of the velocities of the charges, their accelerations and the distance between them. But it should be remembered that in differentiating the retarded potentials use must be made of the forms

$$\frac{\partial t_1}{\partial t_2} = \left[1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}\right], \tag{17}$$

$$\frac{\partial[\psi]}{\partial t} = \frac{1}{\left[1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}\right]}\left[\frac{\partial\psi}{\partial t}\right], \tag{18}$$

$$\frac{\partial[\psi]}{\partial x} = \left[\frac{\partial\psi}{\partial x}\right] - \left[\frac{\partial\psi}{\partial t}\right]\left[\frac{\frac{x}{cR}}{1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}}\right]. \tag{19}$$

The final result may be most compactly expressed as in (15), in which

$$\mathbf{E} = \frac{e_2(1 - \beta_2^2)}{R^3\left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}\right)^3}\left\{\mathbf{R} - \frac{R}{c}\,\mathbf{q}_2 + \frac{\left\{\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\,\mathbf{q}_2\right)\right\} \times \mathbf{R}}{c^2(1 - \beta_2^2)}\right\}, \tag{20}$$

and

$$\mathbf{H} = \frac{e_2(1 - \beta_2^2)}{R^3\left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}\right)^3}\left\{-\frac{\mathbf{R} \times \mathbf{q}_2}{c} + \frac{\mathbf{R} \times \left\{\left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\,\mathbf{q}_2\right)\right] \times \mathbf{R}\right\}}{Rc^2(1 - \beta_2^2)}\right\}. \tag{21}$$

But, since we desire to trace back to their origin the different terms, we will write in detail the different parts. The vector in the last term of (20) may be written

$$\left\{\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\,\mathbf{q}_2\right)\right\} \times \mathbf{R} = (\mathbf{f}_2 \cdot \mathbf{R})\mathbf{R} - R^2\mathbf{f}_2 + \frac{R}{c}(\mathbf{q}_2 \cdot \mathbf{R})f_2 - \frac{R}{c}(\mathbf{q}_2 \cdot \mathbf{R})\mathbf{q}_2. \tag{22}$$

Hence, putting

$$A = \left(1 - \frac{\mathbf{q}_2 \cdot \mathbf{R}}{cR}\right),$$

the value of $\mathbf{E}$ becomes

$$\mathbf{E} = \frac{e_2}{R^3A^3}\left\{\mathbf{R} - \frac{R}{c}\mathbf{q}_2 - \frac{q_2^2}{c^2}\mathbf{R} + \frac{R}{c}\frac{q^2}{c^2}\mathbf{q}_2 + \frac{1}{c^2}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{R} - \frac{R^2}{c^2}\mathbf{f}_2 + \frac{R}{c^3}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{f}_2 - \frac{R}{c^3}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{q}_2\right\}. \tag{23}$$

Returning to the potentials (13) and (14), to find $-\nabla\phi$ and $-\frac{1}{c}\frac{\partial\alpha}{\partial t}$ for substitution in (16), we have

$$-\nabla[\phi] = -[\nabla\phi] + \frac{1}{cRA}\left[\frac{\partial\phi}{\partial t}\right]\mathbf{R} \tag{23a}$$

and

$$-[\nabla\phi] = \frac{e_2}{R^3A^3}\left\{A\left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2\right)\right\}, \tag{24}$$

$$+\frac{1}{cRA}\left[\frac{\partial\phi}{\partial t}\right]\mathbf{R} = \frac{e_2}{R^3A^3}\left\{+\frac{\mathbf{q}_2\cdot\mathbf{R}}{cR}\mathbf{R} + \frac{1}{c^2}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{R} - \frac{q_2^2}{c^2}\mathbf{R}\right\}, \tag{25}$$

$$-\frac{1}{c}\frac{\partial[\alpha]}{\partial t} = \frac{e_2}{R^3A^3}\left\{-\frac{R^2}{c^2}A\mathbf{f}_2 - \frac{1}{c^2}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{q}_2 - \frac{R}{c^3}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{q}_2 + \frac{Rq_2^2}{c^3}\mathbf{q}_2\right\}. \tag{26}$$

But, comparing (15) and (16),

$$\mathbf{E} = -\nabla[\phi] - \frac{1}{c}\frac{\partial[\alpha]}{\partial t}. \tag{27}$$

Hence, by adding (23) and (26) or (24), (25) and (26)

$$\begin{aligned}\mathbf{E} = \frac{e_2}{R^3A^3}\Big\{&\mathbf{R} - \frac{R}{c}\mathbf{q}_2 - \frac{1}{cR}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{R} + \frac{1}{c^2}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{q}_2, \text{ or } -[\nabla\phi]\\ &+\frac{1}{cR}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{R} + \frac{1}{c^2}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{R} - \frac{q_2^2}{c^2}\mathbf{R}, \text{ or } +\frac{1}{cRA}\left[\frac{\partial\phi}{\partial t}\right]\\ &-\frac{R^2}{c^2}\mathbf{f}_2 + \frac{R}{c^3}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{f}_2 - \frac{1}{c^2}(\mathbf{q}_2\cdot\mathbf{R})\mathbf{q}_2\\ &-\frac{R}{c^3}(\mathbf{f}_2\cdot\mathbf{R})\mathbf{q}_2 + \frac{Rq_2^2}{c^3}\mathbf{q}_2, \text{ or } -\frac{1}{c}\frac{\partial[\alpha]}{\partial t}\Big\}.\end{aligned} \tag{28}$$

It is to be observed that two pairs of terms in this cancel out, leaving the result identical with (23) above, which, therefore, completes the derivation of the electric force **E** from the potentials. As an alternative way of writing the fundamental equation for the mechanical force (15) or (16), we may put

$$\mathbf{F} = e_1\left\{-\nabla[\phi] - \frac{1}{c}\frac{\partial[\alpha]}{\partial t} + \frac{1}{c}\mathbf{q}_1\times(\nabla\times[\alpha])\right\}, \tag{16a}$$

where the operations to be performed are according to (17), (18) and (19). Hence we find

$$\mathbf{F} = e_1 \left\{ \begin{array}{l} -\nabla[\phi] \text{ or } -[\nabla\phi] + \frac{1}{cR}\left(\frac{\partial t_2}{\partial t_1}\right)\left[\frac{\partial \phi}{\partial t}\right] \\ -\frac{1}{c}\frac{\partial[\alpha]}{\partial t} \text{ or } -\frac{1}{c}\left(\frac{\partial t_2}{\partial t_1}\right)\left[\frac{\partial \alpha}{\partial t}\right] \\ +\frac{1}{c}\mathbf{q}_1 \times (\nabla \times [\alpha]) \text{ or} \\ \qquad \frac{1}{c}\mathbf{q}_1 \times [\nabla \times \alpha] - \frac{1}{c^2 R}\left(\frac{\partial t_2}{\partial t_1}\right)\left(\mathbf{q}_1 \times \left[\mathbf{R} \times \frac{\partial \alpha}{\partial t}\right]\right) \end{array} \right\}. \quad (16b)$$

Here $[\nabla\phi]$ and $[\nabla \times \alpha]$ involve the differentiation of ϕ and α with respect to the space coördinates but not with respect to the time, and all the other terms involve their differentiation with respect to the time. The force equation is here put into this particular form because it is desirable to separate the differentiations with respect to space and time for reasons that appear in a later section of this paper.

UNIFORM CIRCULAR MOTION.

The above equations are general for any kind of motion of the two charges. We will now introduce the conditions that must be satisfied when the charges each move at constant angular velocity in circular orbits. In a previous communication[1] I have given the solution for the special case of two coaxial orbits only. In dealing with matter in its general form the solution of this problem for the most general possible positions of two circles must be obtained.

To define the circular motion select two sets of rectangular axes i, j, and k for the charge e_1, and i', j' and k' for e_2, taking the origin of the former at the center of the orbit of e_1, and the latter at the center of the orbit of e_2. The k and k' axes are respectively perpendicular to the planes of the orbits of e_1 and e_2, the positive directions being such that the rotation of the electrons as viewed from the positive end of the axis are each in the clockwise sense. In general the planes of the two orbits intersect in some line. This common intersection is selected for the direction of the j- and j'-axes, so that for our purposes the prime may be suppressed, the directions of the two axes being common. The positive direction along the j- and j'-axes is defined by the vector $k \times k'$, which always takes the direction of the j-axis.

Let O_1 and O_2 represent the fixed centers of the orbits of e_1 and e_2, and let $\mathbf{r}_1$ and $\mathbf{r}_2$ be the rotating vectors O_1e_1 and O_2e_2 respectively. These may be defined by the equations

$$\mathbf{r}_1 = a_1(S_1 i + C_1 j), \quad (29)$$

$$\mathbf{r}_2 = a_2(S_2 i' + C_2 j'), \quad (30)$$

[1] Not yet published.

where a_1 and a_2 are the radii, and S and C represent the expressions

$$S_1 = \sin(\omega_1 t + \theta_1), \qquad C_1 = \cos(\omega_1 t + \theta_1), \tag{31}$$

$$S_2 = \sin\left[\omega_2\left(t - \frac{R}{c}\right) + \theta_2\right], \quad C_2 = \cos\left[\omega_2\left(t - \frac{R}{c}\right) + \theta_2\right], \tag{32}$$

in which ω_1 and ω_2 are the angular velocities, and θ_1 and θ_2 the respective phase angles at the time t.

Let $\mathbf{r}$ represent the constant vector from the center O_1 to O_2, and $\mathbf{R}$ the variable vector from e_2 at time $t - R/c$ to e_1 at time t. Then

$$\mathbf{R} = \mathbf{r}_1 - \mathbf{r} - \mathbf{r}_2. \tag{33}$$

The i' and k' axes may be transformed into the i and k axes by the relations

$$i' = (\cos\alpha)i + (\sin\alpha)k, \tag{34}$$

$$k' = -(\sin\alpha)i + (\cos\alpha)k. \tag{35}$$

Denoting by x, y and z the coördinates of the center O_2 referred to O_1 as origin, and writing all quantities in terms of the rectangular unit axes i, j and k, we have

$$\mathbf{r} = xi + yj + zk, \text{ a constant.} \tag{36}$$

$$\mathbf{r}_1 = a_1(S_1 i + C_1 j), \tag{29}$$

$$\mathbf{r}_2 = a_2(S_2 \cos\alpha\, i + C_2 j + S_2 \sin\alpha\, k), \tag{37}$$

$$\mathbf{R} = (-x + a_1S_1 - a_2S_2\cos\alpha)i + (-y + a_1C_1 - a_2C_2)j + (-z - a_2S_2\sin\alpha)k. \tag{38}$$

From this we get the value of R^2

$$R^2 = (r^2 + a_1^2 + a_2^2) + 2(-a_1xS_1 - a_1yC_1 + a_2z\sin\alpha\, S_2 + a_2yC_2 + a_2x\cos\alpha\, S_2 - a_1a_2\cos\alpha\, S_1S_2 - a_1a_2C_1C_2). \tag{39}$$

Denoting by s^2 the first parenthesis in (39), as being quantities independent of the time, and the rest of (39) by us^2, u being a function of the time, we have

$$R^2 = s^2(1 + u), \tag{40}$$

where

$$u = 2/s^2 \text{ (quantities in second parenthesis of (39))}. \tag{41}$$

The velocities and accelerations of the charges may be obtained by differentiation of (29) and (37), giving

$$\mathbf{q}_1 = a_1\omega_1(C_1 i - S_1 j), \tag{42}$$

$$\mathbf{q}_2 = a_2\omega_2(C_2\cos\alpha\, i - S_2 j + C_2\sin\alpha\, k), \tag{43}$$

$$\mathbf{f}_2 = -a_2\omega_2^2(S_2\cos\alpha\, i + C_2 j + S_2\sin\alpha\, k). \tag{44}$$

The above are all the expressions required on the hypothesis of uniform circular motion to find the complete values of **E** and **H** and the mechanical force **F**. The process of finding them is long on account of the large number of terms involved but it seems worth giving in full. We have to find several vectors required by the complex expressions in **E** and **H**, (20) and (21), first

$$\begin{aligned}\mathbf{R} - \frac{R}{c}\mathbf{q}_2 = {} & (-x + a_1S_1 - a_2S_2\cos\alpha - R\beta_2\cos\alpha\, C_2)i \\ & + (-y + a_1C_1 - a_2C_2 + R\beta_2S_2)j \\ & + (-z - a_2S_2\sin\alpha - R\beta_2C_2\sin\alpha)k,\end{aligned} \tag{45}$$

$$\begin{aligned}\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2\right) = {} & -a_2\omega_2^2\{[-zC_2 - R\beta_2\sin\alpha \\ & + yS_2\sin\alpha - a_1S_2C_1\sin\alpha]i + [+(z\cos\alpha \\ & - x\sin\alpha)S_2 + a_1S_1S_2\sin\alpha]j + [-yS_2\cos\alpha \\ & + a_1C_1S_2\cos\alpha + R\beta_2\cos\alpha + xC_2 - a_1C_2S_1]k\},\end{aligned} \tag{46}$$

$$\begin{aligned}\frac{\left[\mathbf{f}_2 \times \left(\mathbf{R} - \frac{R}{c}\mathbf{q}_2\right)\right] \times \mathbf{R}}{C^2(1-\beta_2^2)} = {} & -\frac{a_2\omega_2^2}{c_2(1-\beta_2^2)}\{(-z - a_2S_2\sin\alpha)[(z\cos\alpha \\ & - x\sin\alpha)S_2 + a_1S_1S_2\sin\alpha]i - (-y + a_1C_1 - a_2C_2)[-yS_2\cos\alpha \\ & + a_1C_1S_2\cos\alpha + R\beta_2\cos\alpha + xC_2 - a_1C_2S_1]i + (-x + a_1S_1 \\ & - a_2S_2\cos\alpha)[-yS_2\cos\alpha + a_1C_1S_2\cos\alpha + R\beta_2\cos\alpha + xC_2 \\ & - a_1C_2S_1]j - (-z - a_2S_2\sin\alpha)[-zC_2 - R\beta_2\sin\alpha + yS_2\sin\alpha \\ & - a_1S_2C_1\sin\alpha]j + (-y + a_1C_1 - a_2C_2)[-zC_2 - R\beta_2\sin\alpha \\ & + yS_2\sin\alpha - a_1S_2C_1\sin\alpha]k - (-x + a_1S_1 - a_2S_2\cos\alpha) \\ & - [(z\cos\alpha - x\sin\alpha)S_2 + a_1S_1S_2\sin\alpha]k\}.\end{aligned} \tag{47}$$

The complete part of the force arising from the first two terms $-\nabla\phi - [(1/c)(\partial\alpha/\partial t)]$, or from the first term of (15) may now be found by adding (45) and (47) and multiplying by e_1, whence after multiplying out the quantities in (47), and putting

$$A = 1 - \frac{\mathbf{q}_2\cdot\mathbf{R}}{CR},$$

and separating the terms in various powers of R, we obtain

$$\mathbf{F}_{E_i} = e_{1i}\mathbf{E} = \frac{e_1e_2}{R^2A^3}[-\beta_2C_2\cos\alpha]i \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[-x+a_1S_1-a_2S_2\cos\alpha]i \text{ from (45), the electrostatic part not involving } \beta,$$

$$+\frac{e_1e_2}{R^2A^3}[+\beta_2^3C_2\cos\alpha]i \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[\beta_2^2(+\underline{x}-a_1S_1+a_2S_2\cos\alpha)]i \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}\left[\beta_2^3\cos\alpha\left(-\frac{\underline{y}}{\underline{a_2}}+\frac{a_1}{a_2}C_1-C_2\right)\right]i \text{ from (47)} \qquad (48)$$

$$+\frac{e_1e_2}{R^3A^3}\left\{\frac{\beta_2^2}{a_2}[(z\cos\alpha-x\sin\alpha)zS_2+a_1zS_1S_2\sin\alpha\right.$$

$$\underline{\underline{+a_2zS_2^2\sin\alpha\cos\alpha}}-\underline{\underline{a_2xS_2^2\sin^2\alpha}}+a_1a_2S_1S_2^2\sin^2\alpha$$

$$+y^2S_2\cos\alpha-2a_1yC_1S_2\cos\alpha-xyC_2+a_1yC_2S_1$$

$$+a_1^2C_1^2S_2\cos\alpha+a_1xC_1C_2-a_1^2C_1C_2S_1+a_2yC_2S_2\cos\alpha$$

$$\left.-a_1a_2C_1C_2S_2\cos\alpha-\underline{\underline{a_2xC_2^2}}+a_1a_2C_2^2S_1]\right\}i \text{ from (47)},$$

$$\mathbf{F}_{E_j} = e_{1j}\mathbf{E} = \frac{e_1e_2}{R^2A^3}[+\beta_2S_2]j \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[-y+a_1C_1-a_2C_2]j \text{ from (45) (electrostatic part)}$$

$$+\frac{e_1e_2}{R^2A^3}[-\beta_2^3S_2]j \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[\beta_2^2(\underline{y}-a_1C_1+a_2C_2)]j \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}\left[\beta_2^3\left(\underline{\underline{\frac{x}{a_2}\cos\alpha}}-\frac{a_1}{a_2}S_1\cos\alpha+S_2+\frac{z}{a^2}\sin\alpha\right)\right]j \qquad (49)$$

from (47)

$$+\frac{e_1e_2}{R^3A^3}\left\{\frac{\beta_2^2}{a_2}[-xyS_2\cos\alpha+a_1xC_1S_2\cos\alpha+x^2C_2\right.$$

$$-a_1xC_2S_1+a_1yS_1S_2\cos\alpha-a_1^2S_1C_1S_2\cos\alpha$$

$$-a_1xS_1C_2+a_2^2S_1^2C_2-\underline{\underline{a_2yS_2^2\cos^2\alpha}}$$

$$+a_1a_2C_1S_2^2\cos^2\alpha+a_2xS_2C_2\cos\alpha-a_1a_2S_2C_2S_1\cos\alpha$$

$$+z^2C_2-yzS_2\sin\alpha+a_2zS_2C_2\sin\alpha-\underline{\underline{a_2yS_2^2\sin^2\alpha}}$$

$$\left.+a_1a_2S_2^2C_1\sin^2\alpha+a_2zC_1S_2\sin\alpha]\right\}j \text{ from (47)},$$

$$\mathbf{F}_{E_k} = e_1\mathbf{E}_k = \frac{e_1e_2}{R^2A^3}[-\beta_2C_2\sin\alpha]k \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[-z-a_2S_2\sin\alpha]k \text{ from (45) (the electrostatic part)}$$

$$+\frac{e_1e_2}{R^2A^3}[+\beta_2{}^3C_2\sin\alpha]k \text{ from (45)}$$

$$+\frac{e_1e_2}{R^3A^3}[\beta_2{}^2(\underline{z}+a_2S_2\sin\alpha)]k \text{ from (45)}$$

$$+\frac{e_1e_2}{R^2A^3}\left[\beta_2{}^3\sin\alpha\left(-\frac{\underline{y}}{\underline{a_2}}+\frac{a_1}{a_2}C_1-C_2\right)\right]k \text{ from (47)} \qquad (50)$$

$$+\frac{e_1e_2}{R^3A^3}\left\{\frac{\beta_2{}^2}{a_2}[-yzC_2+y^2S_2\sin\alpha+a_1zC_1C_2\right.$$

$$-2a_1yC_1S_2\sin\alpha+a_1{}^2C_1{}^2S_2\sin\alpha-\underline{\underline{a_2zC_2{}^2}}$$

$$+a_2yC_2S_2\sin\alpha-a_1a_2C_2S_2C_1\sin\alpha$$

$$-(z\cos\alpha-x\sin\alpha)xS_2-a_1xS_1S_2\sin\alpha$$

$$+a_1(z\cos\alpha-x\sin\alpha)S_1S_2+a_1{}^2S_1{}^2S_2\sin\alpha$$

$$\left.-\underline{\underline{a_2S_2{}^2(z\cos\mathfrak{a}-x\sin\mathfrak{a})\cos\mathfrak{a}}}-a_1a_2S_1S_2{}^2\sin\alpha\cos\alpha]\right\}k$$

from (47).

Before proceeding to find the second part of the force $(e_1/c)\mathbf{q}_1\times\mathbf{H}$ in (15) arising from the magnetic component, let us examine the above result arising from the electric force. I had been given to understand on the part of several that these modern, sometimes called exact, equations based upon retarded potentials give rise to no terms varying as the inverse square of the distance, and that the inverse cube[1] of the distance would be the lowest order term to appear. It was with considerable surprise, therefore, that terms of all orders of the inverse powers of the distance appeared in the equations just obtained, because I had formerly made use of approximate forms not involving retarded potentials, and there was no reason to question the correctness of the findings of others.

AVERAGE VALUES.

The above expressions are for the instantaneous force upon electron e_2 due to e_1. To obtain the average over a long time we must average (48), (49) and (50) over a long time. Taking the quantity

$$A = 1-\frac{\mathbf{q}_2\cdot\mathbf{R}}{cR},$$

[1] F. R. Sharpe, PHYS. REV., Sept., 1912, p. 231.

which appears in the coefficients of the equations, as sensibly equal to unity when q_2 is a small quantity compared with c, we obtain different results in averaging dependent upon the relative values of the angular velocities of the two electrons. If the angular velocities are incommensurable, which would be the case generally, we obtain the following average force expressed in a series[1] of inverse powers of r, and including the r^{-1} and r^{-2} terms only.

$$\begin{aligned}\mathbf{F}_i = e^2\beta_2^2 \Big\{ \frac{\beta_2}{a_2} (-\, Y\cos\alpha) r^{-1} + [\tfrac{1}{2}X - \tfrac{1}{2}X\sin^2\alpha + \tfrac{1}{2}Z\sin\alpha\cos\alpha \\ - \tfrac{3}{2}Z^3\sin\alpha\cos\alpha - \tfrac{3}{2}XZ^2\cos^2\alpha + \tfrac{3}{2}XZ^2\sin^2\alpha + \tfrac{3}{2}X^2Z\sin\alpha\cos\alpha \\ - \tfrac{3}{2}Y^2Z\sin\alpha\cos\alpha - \tfrac{3}{2}XY^2\cos^2\alpha + \tfrac{3}{2}XY^2] r^{-2} \cdots \Big\} i,\end{aligned} \tag{51}$$

$$\begin{aligned}\mathbf{F}_j = e^2\beta_2^2 \Big\{ \frac{\beta_2}{a_2} (X\cos\alpha + Z\sin\alpha) r^{-1} + [\tfrac{1}{2}Y + 3XYZ\sin\alpha\cos\alpha \\ + \tfrac{3}{2}X^2Y\cos^2\alpha + \tfrac{3}{2}YZ^2\sin^2\alpha - \tfrac{3}{2}YZ^2 - \tfrac{3}{2}X^2Y] r^{-2} \Big\} j,\end{aligned} \tag{52}$$

$$\begin{aligned}\mathbf{F}_k = e^2\beta_2^2 \Big\{ \frac{\beta_2}{a_2} (-\, Y\sin\alpha) r^{-1} + [\tfrac{1}{2}Z - \tfrac{1}{2}Z\cos^2\alpha + \tfrac{1}{2}X\sin\alpha\cos\alpha \\ - \tfrac{3}{2}Y^2Z\sin^2\alpha - \tfrac{3}{2}XY^2\sin\alpha\cos\alpha + \tfrac{3}{2}XZ^2\sin\alpha\cos\alpha \\ - \tfrac{3}{2}X^2Z\sin^2\alpha + \tfrac{3}{2}X^2Z\cos^2\alpha - \tfrac{3}{2}X^3\sin\alpha\cos\alpha \\ + \tfrac{3}{2}Y^2Z] r^{-2} \cdots \Big\} k.\end{aligned} \tag{53}$$

The letters X, Y and Z have been introduced as equivalent to x/r, y/r and z/r, the direction cosines of the center of the orbit of e_2 referred to the axes i, j and k with origin at O_1. α is the fixed angle between the directions of the axes of rotation of the two electrons, and r the fixed distance between the centers.

Had the two electrons been synchronous with each other so that $\omega_1 = \omega_2$, additional terms involving the constant angle of phase difference, γ, would have appeared, but only in the first power of $\sin\gamma$ and $\cos\gamma$, and, since all such terms cancel out in summing up for a ring of two or more electrons, such as we suppose exist in an atom, we do not obtain

[1] In former papers the letters m, n and v were used instead of a_1, a_2 and r respectively, as denoting the same quantities, the radii of the two orbits and the distances between their centers, the former, however being measured in a small unit a_* instead of in centimeters, and being therefore ratios to this unit. The value of v seldom is less than 100 and the series is evidently rapidly convergent when expressed in inverse powers of v. The convergence is just the same when the equation is written in terms of a and r, although at first sight the convergence is not so evident. The use of the former notation is therefore discarded here as being unnecessary.

any additional terms. It should be remarked that the electrostatic terms in (48), (49) and (50), those which remain when $\beta = 0$, namely

$$\mathbf{F} = \frac{e_1 e_2}{R^3}\{[-x + a_1 S_1 - a_2 S_2 \cos\alpha]i + [-y + a_1 C_1 - a_2 C_2]j + [-z - a_2 S_2 \sin\alpha]k\},$$

have been omitted in obtaining the average force for the reason that their value has been obtained elsewhere.[1] When the effect of the positive charge of the atom is also included so as to get the force between atoms instead of electrons it has been shown that the lowest order terms which make their appearance due to these electrostatic forces is in r^{-6}.

The above result shows that in general the force between two electrons in circular orbits is not in the direction of the line joining the centers of the orbits, but if we resolve the force along the direction or $\mathbf{r}$ the inverse first power terms exactly cancel each other, leaving the force-series to begin with r^{-2} as the lowest order term. To show this multiply (51) by X, (52) by Y and (53) by Z and add to resolve the whole force along $\mathbf{r}$, giving

$$\mathbf{F}_r = e^2\beta_2^2\{\tfrac{1}{2} - \tfrac{1}{2}(-X\sin\alpha + Z\cos\alpha)^2\}r^{-2}. \qquad (54)$$

INTERPRETATION OF THIS RESULT.

This equation expresses that portion of the total force between two electrons revolving in circular orbits resolved along the line joining centers arising from the electric force $\mathbf{E}$, but not including the electrostatic terms as above explained. If we obtain in a similar manner the portion arising from the magnetic force $\mathbf{H}$, another set of inverse square terms is obtained, but the coefficient of all such terms is in the higher orders of the small quantity β, and they are, therefore, negligible in comparison with the terms in (54) arising from the electric force. This equation, therefore, expresses the whole force to a very close degree of approximation between two electrons in circular orbits having incommensurable velocities resolved along the line of centers, when the distance between the centers is very large compared to the radius of either orbit, because the terms in higher powers, r^{-3}, r^{-4}, etc., become inappreciable if the distance r is large enough.

In case the angular velocities are equal, as above explained, this equation applies to rings of electrons when multiplied by the numbers in the rings, and it does not matter in the case of rings whether the velocities are synchronous or non-synchronous. There is no limit to the distances at which these forces are felt, and the equation thus sug-

[1] The average of the electrostatic forces for atom on atom is given by the r^{-6} terms in equations (23), (24) and (25), p. 755, Phil. Mag., June, 1915.

gests that this may be the gravitational force because it varies inversely as the square of the distance. On this account it is worth making a critical examination of this equation. The general appearance is right for the gravitational force for several reasons. First, the force is always an attraction independent of the directions of the axes. Heretofore, the forces between atoms as yet obtained in former papers have depended almost entirely upon the directions of the axes of rotation, being either an attraction or a repulsion according to their directions. We may show that this is not the case with equation (54). Let us designate the ring of electrons, of which e_2 is one, as atom A_2, and the ring, of which e_1 is one, as atom A_1. Imagine that the centers of the two atoms are fixed in space, and suppose that the axis of rotation of A_2 is prolonged, and that a perpendicular line from the center of A_1 is drawn to it. The quantities within the brace in (54) then represent $\frac{1}{2}$ the square of the ratio of this perpendicular line to the fixed distance r. The part $-x \sin\alpha + z\cos\alpha$ is the distance from A_2 along its axis to the foot of this perpendicular line, hence $r^2 - (-x \sin\alpha + z\cos\alpha)^2$ is the square of the perpendicular line, and the square of the ratio of this perpendicular to the distance r is $1 - (-X \sin\alpha + Z\cos\alpha)^2$, which quantity appears in (54).

When the distance from A_2 to the foot of this perpendicular vanishes, the whole force reduces to

$$\mathbf{F} = \tfrac{1}{2}e^2\beta_2^2 r^{-2}. \tag{55}$$

It will thus vanish when the axis of A_2 is perpendicular to the line $\mathbf{r}$, the atom A_1 then lying in the equatorial plane of A_2. If we apply the theory of probabilities to the case, and suppose that the body, of which A_2 is but a single atom, is either a gas, liquid, or homogeneous solid without crystalline structure, it is then just as likely that the positive pole of any axis will point toward A_1 as that a negative pole will do so, and the position of the average one of a large number of atoms must be such that A_1 lies in the equatorial plane of the average atom in the second mass, and for such an atom the force is that given by (55).

Another way of looking at the matter is to observe that the sum of the squares of the perpendicular lines drawn from the mass in which A_1 is one atom, to the various axes of rotation produced from the atoms in the second mass in no way depends upon the orientation of the axes in the first mass, so that we are at liberty to orient the first mass as we please without affecting the attraction due to the second mass. If this is true of mass No. 1 it must also be true of mass No. 2, for we may make the calculation the other way about interchanging the masses.

A third consideration on the same subject is to fix the attention upon a single atom A_1, upon which the force is to be obtained. The quantities

z or Z are then approximately fixed and invariant for all atoms in mass No. 2. The independent variables are x or X and α. If we seek for the average values of X and α on the theory of probabilities for a large number of atoms in mass No. 2, the average value of α is 60°, and the average value of X is zero. Hence the total force of all atoms in mass No. 2 on the single atom A_1, when divided by the number of atoms in mass No. 2, is

$$\mathbf{F} = \tfrac{1}{2}e^2\beta_2^2(1 - \tfrac{1}{4}Z^2)r^{-2}. \qquad (56)$$

If we next consider other atoms in mass No. 1, the average value of Z is zero, and the total force of mass No. 2 upon mass No. 1 divided by the product of the numbers of atoms in the two masses, provided the atoms are of the same kind, is the same as equation (55) above.

ATTRACTION BETWEEN CRYSTALS.

The question now naturally arises as to the gravitational attraction between crystals. Mackenzie[1] has shown experimentally in a rather convincing manner that the variation in the attraction between two crystals of calcspar when oriented in all possible directions is not greater than one part in about 500, or about .2 per cent. This seems to indicate that there is no variation whatever to a first order approximation, and the proper theory of gravitation should give a reason for this. Before we can proceed to examine this question it is necessary to know just how the axes of rotation are directed in any given crystal. I am not acquainted with any investigations of others upon this subject, and the only crystals as yet investigated belong to the cubic or isometric system, but the determination of the directions of the axes of rotation rests upon a very simple conception, namely, that there is a tendency on the part of any atom to turn the axis of any other atom in the crystal so as to become parallel with itself, and, when a second atom has its axis parallel to the first, the turning moment vanishes. This alone is sufficient to determine the directions of the axes of rotation in a crystal, because the sum of all the turning moments acting upon any atom due to all the other surrounding atoms must vanish. This has been shown to be true for the arrangement of the axes in about twenty forms of crystals each belonging to the cubic system. In each case there are but four different directions to the axes of the various atoms making up the whole crystal, and these are parallel to the four medial lines of a regular tetrahedron, or which is the same, to the four diagonals of a cube. These lines make equal angles with each other, about 109°, or exactly so that $\cos\alpha = -\frac{1}{3}$ and $\sin\alpha = \frac{2}{3}\sqrt{2}$.

[1] A. S. Mackenzie, "On the Attraction of Crystalline and Isotropic Masses at Small Distances," PHYS. REV., Vol. II., Mar.–Apr., 1895, p. 321.

Let us calculate the attractive force between two small diamonds, say, at a great distance apart compared with their size by means of (54). It is necessary to show that the attraction is independent of the orientation of the diamonds if it is the gravitational force. We will calculate the force exerted by the second mass upon the first. Draw four lines parallel to the four directions of the axes of all the atoms in diamond No. 2 through the center of its mass, and then draw a perpendicular line from the center of diamond No. 1 upon each of the four lines thus drawn. Since the whole diamond is made up of equal numbers of atoms with axes parallel to these four directions, it is sufficient to find the attraction of a single group of four atoms with axes, one in each of these four directions, and then to multiply the result by the number of such groups. If we can show that one such group exerts an attraction which is independent of the orientation of the group with respect to the position of the first diamond we shall have proved the case. This is equivalent to saying that the sum of the squares of the four perpendicular lines from diamond No. 1 upon the four different axes of the group in diamond No. 2 shall be constant for a fixed distance, r, between the diamonds. This leads to the statement of a geometrical proposition which, if true, proves the case, as follows.

If through any point four lines be drawn making equal angles each with any other, and, if from a second point at a distance r from the first point four perpendiculars be drawn one to each of the said four lines, then the sum of the squares of these perpendiculars is constant for all points at the same distance from the first point. The locus of the second point is the surface of a sphere with the first point as center.

The truth or falsity of this proposition was unknown at the time of arriving at the conclusion that it must be true if this is the proper form for the gravitational force. It has since been proved to be true.[1]

Another way of looking at the matter is to show that the force in (54) in no way depends upon the orientation of diamond No. 1, for the perpendiculars from No. 1 upon the axes of No. 2 would not be changed by turning the diamond No. 1. If we calculate the force the other way about it is equally evident that we should be able to turn No. 2 in any direction without changing the force.

THE MAGNITUDE OF THE ATTRACTION.

To calculate the magnitude of the attraction between two diamonds let us place the second diamond so that one of the four axes of the atoms

[1] The truth of this proposition has been established by Prof. F. W. Owens independently, though it is possible that it may have been published somewhere.

in it passes through diamond No. 1, since it makes no difference in the force how they are oriented. The perpendicular upon this axis is then zero, and those upon the other three axes are equal to each other. This perpendicular is $\frac{2}{3}\sqrt{2}r$, and the distance from A_2 to the foot of it is $\frac{1}{3}r$. The sum of the squares of the four perpendiculars is then $\frac{8}{3}r^2$. Hence the total force exerted by a group of four electrons, one in each of four atoms composing a group in diamond No. 2, upon a single electron in diamond No. 1 is

$$\mathbf{F} = \tfrac{4}{3}e^2\beta_2^2 r^{-2}. \tag{57}$$

Before we can calculate numerically the total force with which one diamond attracts the other in order to compare the magnitude with that derived from the gravitational constant, it is necessary to know more about the structure of the carbon atom. The diamond has been selected for this purpose because there is but a single kind of atom in it, and the atomic weight of carbon is as small as that of any atom we know which forms crystals without other elements. From previous investigations it is thought that the carbon atom has twelve electrons, consisting of a ring of 8 within which is a ring of 4, and we will proceed with this form of atom. Recently it has been shown that a ring of eight electrons, based upon the fundamental equations (1) to (21) above, which forms the basis of the present discussion, must have a linear velocity so as to make $\beta = .012$ approximately, irrespective of the radius of the orbit. It has also been shown that the ring of four electrons within the eight will not affect the speed of the eight in an appreciable degree. If the ring of four were by itself, it would have a speed such that $\beta = .00846$, but, when within the ring of eight, the mutual forces may bring the speed of the inside ring to the same angular velocity as the outside ring, which really controls the atom. However this may be, it will appear that the inside ring of four has but little effect upon the whole attraction. It has been calculated that the radius of the inside ring is .402 part of that of the outside ring, and, assuming the angular velocity the same as the outside ring, $\beta = .402 \times .012 = .004824$ for the inside ring. For the outside ring $\beta^2 = 1.44 \times 10^{-4}$, and for the inside ring $\beta^2 = .2328 \times 10^{-4}$. To get the sum of β^2 for the carbon atom, multiply the former by 8 and the latter by 4 and add, giving

$$\sum_p \beta^2 = (11.52 + .93) \times 10^{-4} = 12.45 \times 10^{-4},$$

a value not much greater than for the outside ring alone.

To obtain the average pull between one atom in diamond No. 1 and one in No. 2, multiply (57) by 12 for the number of electrons in atom A_1,

and put for β_2^2 the $\sum_P \beta^2$ for the atom A_2, and divide by 4 because (57) is for a group of four electrons. We thus find the average attraction between two carbon atoms at a distance of 1 cm. apart in the diamond to be

$$\mathbf{F} = 4 \times 12.45 \times 10^{-4} e^2 = 49.8 \times 10^{-4} e^2. \tag{58}$$

Taking the value of e in electrostatic units as 4.77×10^{-10}, we find

$$\mathbf{F} = 1134 \times 10^{-24} \text{ dynes.} \tag{59}$$

Let us next assume that the two diamond crystals are cubes 10^{-6} cm. on an edge, and that the distance between their centers of mass is 1 cm. The number of atoms that such a cube with edge a contains is $2\sqrt{2}(a^3/l^3)$ where $l = 2.528 \times 10^{-8}$ cm., this being the edge of the elementary tetrahedron. If N is the number of atoms in this cube, when $a = 10^{-6}$ cm.,

$$N = \frac{2\sqrt{2} \times 10^{-18}}{16.16 \times 10^{-24}} = 175{,}000. \tag{60}$$

Hence the total attraction between the two cubes is N^2 times (59), and

$$\mathbf{F} = .03062 \times 10^{12} \times 1134 \times 10^{-24} = 34.72 \times 10^{-12} \text{ dynes.} \tag{61}$$

THE GRAVITATIONAL ATTRACTION.

This value may now be compared with the gravitational pull between these diamonds as calculated from the well-known gravitational constant. The force at 1 cm. distance is

$$\mathbf{F} = 666 \times 10^{-10} m^2,$$

where m is the mass of one cube. Since m = density $\times$ volume $= 3.51 \times 10^{-18}$, we have

$$\mathbf{F} = 666 \times 10^{-10} \times 3.51^2 \times 10^{-36} = 82.1 \times 10^{-44} \text{ dynes.} \tag{62}$$

A comparison between (61) and (62) shows that the theoretical force is 4.23×10^{31} times the actual gravitational force between the same pieces of matter. In other words the calculated pull between the two diamonds is immensely greater than any existing force acting between them. The order of magnitude is so greatly different that there is no question that the magnitude of the calculated force is in error somewhere. Let us examine the case more closely. The only two hypotheses which have been used in obtaining an absurdly large value for the force between two pieces of matter are, first, that the atoms are composed of electrons revolving at uniform velocities in circles: and, second, that the accepted electromagnetic equations are applicable to these electrons. Unless we reject both hypotheses we are forced to decide between the two horns

of this dilemma. We either have to abandon the idea that the electrons are revolving in circles in the atoms, or that the present form of the electromagnetic equations apply to the electrons in atoms. There is little doubt as to which of the alternatives to choose. We will not yet abandon the idea that the electrons revolve in circles, and are driven to the conclusion that the present form of the equations does not apply to the electrons in the atoms. This is the position that the world of physicists has already taken, but not on these grounds. Some have, perhaps, gone a step too far and settled down to the conviction that we must get on somehow without any electromagnetic equations as far as the atoms are concerned. This does not seem to be a necessary alternative when we realize the very great difficulties which the derivation of these equations presents.

The value of the force derived above has the right look in so many respects that there seems to be some hope that this will open the way to a revision of the present form of the equations so as not only to meet all demands already made upon them but so as to include the gravitational force as well. On this account the derivation of these equations has been given here in some detail. Things look as if some important factor had been omitted somewhere which changes the magnitude of the force but not its general characteristics. A search has been made for such a factor and the place to put it without altering other results that are already satisfactorily interpreted by the equations in their present form. In other words it is to be a factor which affects the gravitational or inverse square terms to a large extent, and some of the other terms to a small extent.

If equation (54), the calculated expression for the gravitational pull, were multiplied by a factor $\frac{1}{4} \times .898 \times 10^{-27} \beta_1^2$, where β_1 is the velocity of the first electron in terms of that of light, we obtain the correct value of the gravitational force within about 1 per cent. It seems very natural that the velocity of the first electron as well as that of the second should come in to the final result, but in the present form of the electric force, from which (54) is derived, β_1 fails to appear.

Making the calculation for the two diamonds after introducing this factor we would have obtained instead of (57)

$$\mathbf{F} = \tfrac{1}{3} \times .898 \times 10^{-27} \beta_1^2 \beta_2^2 e^2 r^{-2} \qquad (67)$$

and instead of (58), writing $\sum_P \beta_1^2$ for atom A_1 instead of 12, the number of electrons in A_1,

$$\mathbf{F} = \tfrac{1}{12} \times .898 \times 10^{-27} \times 12.45^2 \times 10^{-8} e^2 = 11.6 \times 10^{-35} e^2, \qquad (68)$$

and instead of (61)

$$\mathbf{F} = 80.9 \times 10^{-44} \text{ dynes.} \tag{69}$$

This is the revised value of the attraction between the two diamonds to be compared with the gravitational pull (62), namely 82.1×10^{-44}. This value is in very close agreement with the calculated value.

It is to be remarked that the number $.898 \times 10^{-27}$, which is part of the factor that has been introduced, is the numerical value of the mass of the electron at slow velocities, and, hence, the total factor is not merely a numeric but has a real physical significance, being a property of the electron itself. If the factor had no such value it would not mean so much because some factor could of course be found to correct the final result, but that it has a physical meaning seems very significant, and especially so because it is a property of the electron itself. The revised form of the gravitational terms in (54) may then be put into the following symmetrical form,

$$\mathbf{F} = \frac{m_0}{k} e_1 e_2 \frac{\beta_1^2}{2} \frac{\beta_2^2}{2} \{\tfrac{1}{2} - \tfrac{1}{2}(-X \sin\alpha + Z \cos\alpha)^2\} r^{-2}. \tag{70}$$

If the electron is that of Lorentz the mass at slow velocities may be expressed in terms of the charge as follows

$$m_0 = \frac{4e^2}{5c^2 a}, \tag{71}$$

and the transverse mass at any velocity as

$$m = m_0(1 - \beta^2)^{-1/2}, \tag{72}$$

and longitudinal mass as

$$m = m_0(1 - \beta^2)^{-3/2}. \tag{73}$$

In the modern electromagnetic theory the quantities k and μ the specific inductive capacity and the magnetic permeability are usually suppressed. The product only is known $\mu k = 1/c^2$, but not the dimensions of each separately, and until these are known it seems very doubtful that it is correct to write any expression without either of these factors being present even in the common mechanical units such as force, energy or mass. I have, therefore, introduced the k into equation (70) in order to keep the dimensions of the right hand member the same as that of force, letting the unknown dimensions of k compensate for the new mass factor. Using the Lorentz mass (71) for slow velocities, we may write (70), where e is expressed in electrostatic units,

$$\mathbf{F} = \frac{e^4 \beta_1^2 \beta_2^2}{5ac^2 k} [\tfrac{1}{2} - \tfrac{1}{2}(-X \sin\alpha + Z \cos\alpha)^2] r^{-2}, \tag{74}$$

or in electromagnetic units,

$$\mathbf{F} = \frac{\mu e^4 \beta_1^2 \beta_2^2}{5a}[\tfrac{1}{2} - \tfrac{1}{2}(-X \sin\alpha + Z\cos\alpha)^2]r^{-2}. \qquad (75)$$

Comparing the dimensions of the quantities on the right with that of force, we find from (75), in electromagnetic units in the dimensional form,

$$MLT^{-2} = \mu(M^{1/2}L^{1/2}\mu^{-1/2})^4 L^{-3} = \mu^{-1}M^2L^{-1}. \qquad (75)$$

Hence

$$M = \mu L^2 T^{-2} \text{ in the electromagnetic system,} \qquad (76)$$

and similarly from (74) with electrostatic units

$$M = k^{-1} \text{ in the electrostatic system.} \qquad (77)$$

In these dimensional forms β is a numeric, being a ratio of two velocities, and does not appear.

Taking these as the dimensions of mass on the two systems, a new system of dimensions becomes possible in which mass does not appear, there being but three dimensions L, T and k or μ instead of four L, M, T and k or μ in the present systems. A reduction of a few of the more common units according to this new system are given in the following table, those units not involving mass on the old systems remaining unchanged.

Name of Unit.	Electrostatic Old.	Electromagnetic Old.	Electrostatic New.	Electromagnetic New.
Quantity of electricity....	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}k^{\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}\mu^{-\frac{1}{2}}$	$L^{\frac{3}{2}}T^{-1}$	$L^{\frac{3}{2}}T^{-1}$
Quantity of magnetism...	$M^{\frac{1}{2}}L^{\frac{1}{2}}k^{-\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-1}\mu^{\frac{1}{2}}$	$L^{\frac{1}{2}}k^{-1}$	$L^{\frac{3}{2}}T^{-2}\mu$
Electric potential and E.M.F.....	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}k^{-\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}\mu^{\frac{1}{2}}$	$L^{\frac{1}{2}}T^{-1}k^{-1}$	$L^{\frac{3}{2}}T^{-3}\mu$
Electric current.....	$M^{\frac{1}{2}}L^{\frac{3}{2}}T^{-2}k^{\frac{1}{2}}$	$M^{\frac{1}{2}}L^{\frac{1}{2}}T^{-1}\mu^{-\frac{1}{2}}$	$L^{\frac{3}{2}}T^{-2}$	$L^{\frac{3}{2}}T^{-2}$
Resistance.....	$L^{-1}T\ k^{-1}$	$L\ T^{-1}\mu$	$L^{-1}T\ k^{-1}$	$L\ T^{-1}\mu$
Coef. of self-induction....	$L^{-1}T^2\ k^{-1}$	$L\ \mu$	$L^{-1}T^2\ k^{-1}$	$L\ \mu$
Capacity of a condenser..	$L\ k$	$L^{-1}T^2\ \mu^{-1}$	$L\ k$	$L^{-1}T^2\ \mu^{-1}$
Energy.....	$M\ L^2\ T^{-2}$	$M\ L^2\ T^{-2}$	$L^2\ T^{-2}k^{-1}$	$L^4\ T^{-4}\mu$
Mass.....	M	M	k^{-1}	$L^2\ T^{-2}\mu$
Force.....	$M\ L\ T^{-2}$	$M\ L\ T^{-2}$	$L\ T^{-2}k^{-1}$	$L^3\ T^{-4}\mu$

The Origin of the Gravitational Terms.

Let us endeavor to trace to their origin the terms which contributed to the gravitational expression (54). These came immediately from (51), (52) and (53), and these again from the process of averaging (48), (49) and (50). In these latter equations the only terms which have contributed anything to the r^{-2} terms, that is, to (54), are doubly underscored. In the i-component there are five such terms only, one of

which, $(e_1e_2/R^3A^3)\beta_2^2x$, comes from (45), and the rest from (47). This one term is traceable back to the term

$$-\frac{q_2^2}{c^2}\mathbf{R} = -\beta_2^2\mathbf{R},$$

the last term in second line of (28), arising from the differentiation of the scalar potential with respect to the time $[\partial\phi/\partial t]$. All the rest of the terms are traceable to (47) and thence to (22), which terms appear in (28) either as the differentiation of ϕ or α with respect to the time, but not with respect to the space coördinates.

The search thus seems to lead back to two possible sources, either of which, if slightly modified, might have given results more in conformity with several physical phenomena. First, the imposed condition

$$\text{div. } \alpha + \frac{1}{c}\dot{\phi} = 0, \tag{78}$$

which partly defines the potentials ϕ and α. As Schott remarks on page 3 of his "Electromagnetic Radiation," "Thus α is not completely determined, and a third condition may be imposed upon it. Thus a singly infinite series of different functions α are possible, but *only two have been used.*" The first condition to have been used was div. $\alpha = 0$, and the second one equation (6) above, which assumption is the basis of the modern theory.

Second, the fundamental equation attributed to Larmor and Lorentz,

$$\mathbf{F} = e_1\left(\mathbf{E} + \frac{1}{c}\mathbf{q}_1 \times \mathbf{H}\right), \tag{79}$$

by which the mechanical force is derived from the above potentials. This equation expresses an assumed connection between charge and aether. A modification of the forms of either of these assumptions is possible, and seems justifiable, without departing from the more fundamental conceptions of an electromagnetic theory. It must have been necessary originally to make these assumed forms conform to experience so far as then known, and as revealed by physical phenomena. Is it not possible that some new phenomena have now been found which are in slight disagreement with the original forms assumed? It seems natural to expect to have to modify these forms when some new phenomena not originally included in the scheme is found. And, similarly, it is difficult to imagine any way to arrive at correct forms except as these are indicated by additional phenomena. This seems to be a logical method of arriving finally at an all-embracing electromagnetic theory.

If we hazard the guess that the form (6) defining the potentials is

correct, and attempt to alter (15) so as to make these potentials give the proper mechanical force, this process seems to require that a factor be introduced into those parts which involve differentiation with respect to the time. The form (16*b*) is the equivalent of (15), and in this the factor is required in three of the terms but not in two, namely neither in $-[\nabla\phi]$ nor in $\frac{1}{c}q_1 \times [\nabla \times \alpha]$.

A difficulty apparently arises from the fact that, in the current method of attacking the problem by resorting to the potentials, the potential is found at a stationary point in the aether due to a second distant charge in motion, and this potential is then applied to a first charge moving *through the point* at which the potential is found. The only way by which the motion of the first charge contributes anything to the mechanical force is then through the second term of (15), and this does not seem to be sufficient. In some manner the mechanical force seems to demand an integration extended over the volumes of both charges, and one of them at least, being in motion through the region where the force is wanted, should be a function of the time, the same charge not occupying the same element of volume constantly. This integration does not come into the derivation of the potential at a point, and so escapes entirely unless it is included separately in such a relation as (15) connecting the potential with the mechanical force.

It is with regret that no fundamental solution is as yet found that will introduce the required changes in the mechanical force, and the subject has been presented at some length in the hope that some one will have some solution to suggest.

OTHER ATOMIC PHENOMENA.

To show that these changes required for the gravitational force are in complete harmony with other phenomena which also depend upon the same modified electromagnetic equations, we consider below three distinct properties of the diamond which depend upon the higher order terms of the force-series, which become effective at molecular distances in distinction to the great distances heretofore employed. The same treatment applies, of course, to other substances, but it is not included here. They are: (1) The speed of electrons in rings, such as those composing the atoms; (2) the structure of the diamond, and determination of the radius of the orbit of the electrons in the carbon atom; (3) the determination of the bulk-modulus and the force required to produce a given contraction in the volume per unit volume.

Let us first write the complete forms of **E** and **H** as modified by the

requirements for the gravitational terms above discussed. These may be compared with (20) and (21) above.

$$\mathbf{E}=\frac{e_2(1-\frac{1}{4}m\beta_1^2\beta_2^2)}{R^3A^3}\left\{\mathbf{R}-\frac{R}{c}\mathbf{q}_2+\frac{\frac{1}{4}m\beta_1^2\left\{\left[\mathbf{f}_2\times\left(\mathbf{R}-\frac{R}{c}\mathbf{q}_2\right)\right]\times\mathbf{R}\right\}}{c^2(1-\frac{1}{4}m\beta_1^2\beta_2^2)}\right\}, \tag{80}$$

$$\mathbf{H}=\frac{e_2(1-\frac{1}{4}m\beta_1^2\beta_2^2)}{R^3A^3}\left\{-\frac{\mathbf{R}\times\mathbf{q}_2}{c}+\frac{\frac{1}{4}m\beta_1^2\left\{\mathbf{R}\times\left[\mathbf{f}_2\times\left(\mathbf{R}-\frac{R}{c}\mathbf{q}_2\right)\right]\times\mathbf{R}\right\}}{c^2R(1-\frac{1}{4}m\beta_1^2\beta_2^2)}\right\}. \tag{81}$$

If we follow the same plan with these equations as was done with (20) and (21) above and find the i, j and k components of the mechanical force as in (48), (49) and (50), we obtain a similar expression for $\mathbf{F} = e_1\mathbf{E}_i$ term for term as in (48), (49) and (50), except that most of the terms have an additional factor $\frac{1}{4}m_0$. The only terms which do not have this factor are those in the first two lines of these equations, as follows for the $\mathbf{E}$ part

$$\mathbf{F}_{E_i}=e_1\mathbf{E}_i=\frac{e_1e_2}{R^3A^3}[-x+a_1S_1-a_2S_2\cos\alpha]i \text{ (electrostatic part)}$$
$$+\frac{e_1e_2}{R^2A^3}[-\beta_2C_2\cos\alpha]i, \tag{82}$$

$$\mathbf{F}_{E_j}=e_1\mathbf{E}_j=\frac{e_1e_2}{R^3A^3}[-y+a_1C_1-a_2C_2]j \text{ (electrostatic part)}$$
$$+\frac{e_1e_2}{R^2A^3}[+\beta_2S_2]j, \tag{83}$$

$$\mathbf{F}_{E_k}=e_1\mathbf{E}_k=\frac{e_1e_2}{R^3A^3}[-z-a_2S_2\sin\alpha]k \text{ (electrostatic part)}$$
$$+\frac{e_1e_2}{R^2A^3}[-\beta_2C_2\sin\alpha]k. \tag{84}$$

These three equations are equivalent to

$$\mathbf{F}=e_1\mathbf{E}=\frac{e_1e_2}{R^3A^3}\left\{\mathbf{R}-\frac{R}{c}\mathbf{q}_2\right\}. \tag{85}$$

To get the complete force we have to add to this the terms having the very small factor less than 10^{-30}, the addition of which makes no appreciable difference in (85). If, therefore, we contemplate using these equations at small distances where any one of the terms without the small factor is effective, it becomes the whole force, and we might have put $m = 0$ just as well in the forms (80) and (81) so far as these terms

are concerned. Doing this would have given (85) directly, and instead of (81) we would have

$$\mathbf{H} = \frac{e_2}{R^3 A^3}\left[-\frac{\mathbf{R}\times\mathbf{q}_2}{c}\right]. \tag{86}$$

Had we continued the process of averaging the instantaneous force as was done with (48), (49) and (50) to obtain (51), (52) and (53) expressed as an infinite series in terms of $1/r$, we should have obtained just the expressions given in (51), (52) and (53), which only include the r^{-2} terms plus other terms in higher powers which are there omitted. Now, omitting the electrostatic part as before, the terms in (85) and (86) do not appear at all in the r^{-2} terms, and make their first appearance in the series in the higher powers only. At molecular distances these higher power terms arising from (85) and (86), being the β-terms which do not have the small factor, represent the whole force, the terms in (51), (52) and (53) being entirely ineffective, as well as those in higher powers which contain the small factor.

These equations (85) and (86) will now be applied to the case of a single ring of electrons, which has been calculated elsewhere[1] by means of the original unmodified equations (20) and (21), in order to make a comparison.

It is required to find the force exerted by a second electron in the same orbit as the first, resolved along the tangent line to the orbit of the first. It may be shown that $\mathbf{H}$ has no component along the tangent line, the vector $\mathbf{R}\times\mathbf{q}_2$ in (86) being perpendicular to the plane of the orbit, so that the electric force controls the ring. When the centers of the orbits coincide, the values of $\mathbf{r}$ and its components x, y and z in (36) are zero. And, since the axes of rotation are common, $\alpha = 0$. The radii $a_1 = a_2 = a$, and $\beta_1 = \beta_2 = \beta$. Hence the value of $\mathbf{R}$ in (38) becomes

$$\mathbf{R} = a\{(S_1 - S_2)i + (C_1 - C_2)j\} \tag{87}$$

and

$$\mathbf{q}_2 = a\omega(C_2 i - S_2 j). \tag{88}$$

Hence

$$\mathbf{F} = e_1\mathbf{E} = \frac{e_1 e_2 a}{R^3 A^3}\left\{\left(S_1 - S_2 - \frac{R\beta}{a}C_2\right)i + \left(C_1 - C_2 + \frac{R\beta}{a}S_2\right)j\right\}. \tag{89}$$

To resolve this force along the tangent line $\tau = S_1 j - C_1 i$, take the dot product with (89), and find

$$\underset{\text{tan}}{\mathbf{F}} = \frac{e_1 e_2 a}{R^3 A^3}\left\{C_1 S_2 - C_2 S_1 + \frac{R\beta}{a}(C_1 C_2 + S_1 S_2)\right\}.$$

[1] A. C. Crehore, "On the Cause of the Revolution of a Ring of Electrons." As yet unpublished.

Since

$$C_1S_2 - C_2S_1 = \sin\gamma, \quad \text{and} \quad C_1C_2 + S_1S_2 = \cos\gamma, \tag{90}$$

where $\gamma = \theta_2 - \theta_1$, the constant angle of phase difference (see (31) and (32) above), we have

$$\underset{\text{tan}}{\mathbf{F}} = \frac{e_1e_2a}{R^5A^3}\left\{\sin\gamma + \frac{R\beta}{a}\cos\gamma\right\}. \tag{91}$$

This is for clockwise rotation of the electrons. For counterclockwise rotation reverse the sign.

To compare this result from the modified equations with that before obtained from the original forms, we give the former result

$$\underset{\text{tan}}{\mathbf{F}} = \frac{e_1e_2a}{R^3A^3}\left\{(1 - 2\beta^2)\sin\gamma + \frac{R\beta}{a}\cos\gamma - \frac{R\beta}{a}\beta^2 + \beta^2\sin\gamma\cos\gamma\right\}. \tag{92}$$

In each of these results we have entirely neglected the force which an electron exerts upon itself which has been the subject of much discussion. That is to say, we have neglected the radiation of energy. The justification for this is to be found in the facts. The supply of this energy has been accounted for by some in supposing a slow secular change in the mass of the electron. In the words of Schott[1] "the change of mass need only take place very slowly provided that the velocity of the ring shall be small enough. In order that it should not contradict our experience as to the constancy of the properties of electrons and atoms, it is sufficient that $\dot{m}/m$ be less than, let us say 10^{-15}, or one thirty millionth part per annum, but of course it might be very much smaller." Since we are proceeding to use modified electromagnetic equations, being led to do this for the reasons given above, it seems most probable that the ordinary calculation of the radiation will have to be revised, and may turn out to be of a much smaller order than heretofore supposed. At any rate, experience shows that it is negligible on account of the permanency of atoms, and, if it is clearly stated that the radiation is being neglected here, this should not alter the value of the results obtained if they also agree with experience.

Due to the fact that the orbit is a circle, the equations (91) or (92) require definite numerical values for β for a definite number of electrons in the ring. The additional equation required from the properties of the circle, which makes the solution a definite numerical value, is

$$R^2 = 2a^2(1 - \cos\gamma).$$

Without making the calculation here we will content ourselves with the statement that very approximately the same numerical values of

[1] Electromagnetic Radiation, p. 191.

β are obtained from either the original equation (92), or the modified form (91), with the single exception of a ring of only two electrons. In the case of a ring of four electrons, the value is $\beta = .00846$, and in the case of a ring of eight $\beta = .012$. The equilibrium is very approximately determined by the two electrons adjacent to that on which the force is obtained, one in advance and the other behind the selected electron. The effect of all the other electrons in the ring is comparatively small. To give an idea of the degree of difference due to the use of (92) or (91), when $\beta = .001$ in the case of a ring of four electrons, the sum of the forces due to the two adjacent electrons along the tangent line is $.000,007,247,37(e^2/a^2)$ using (92), and $.000,007,247,06(e^2/a^2)$ using (91), a difference of one part in about 10,000. Adding the force due to the third or opposite electron in the ring does not affect this figure.

The exception noted above in the case of a ring of two electrons is such as to completely change the result. If we use the unmodified equation (92), the force on the one electron due to the other has the opposite sign to that obtained from (91). In the former case there is no equilibrium, or rather it is unstable, but in the latter case the equilibrium is stable and the value of β must be zero. That is to say, this form of the equation permits the two electrons to remain at rest, and, if started in motion due to any cause, the mutual forces bring them to rest again when the disturbance ceases.

CRYSTAL STRUCTURE.

We shall now proceed to derive the average force exerted by one electron e_2 on another e_1 including the higher powers of the force-series, which in (51), (52) and (53) terminated with the r^{-2} term, in order to apply it to the formation of molecules, crystals being regarded as one type of molecule. As pointed out above, the force series, obtained from the complete forms (80) and (81), is the same for our purposes as that obtained from the shorter forms (85) and (86). From these we have for circular motion, omitting the electrostatic part $\mathbf{R}$ in (85) as before,

$$\mathbf{E} = \frac{e_2}{R^2A^3}\left\{-\frac{R}{c}\mathbf{q}_2\right\} = -\frac{e_2\beta_2}{R^2A^3}\{C_2\cos\alpha\, i - S_2 j + C_2\sin\alpha\, k\}, \quad (93)$$

$$\begin{aligned}\mathbf{H} = \frac{e_2}{R^2A^3}\left\{-\frac{\mathbf{R}\times\mathbf{q}_2}{c}\right\} = -\frac{e_2\beta_2}{R^2A^3}\{&[(-x + a_1S_1 - a_2S_2\cos\alpha)i \\ &+ (-y + a_1C_1 - a_2C_2)j + (-z - a_2S_2\sin\alpha)k] \\ &\times [C_2\cos\alpha\, i - S_2 j + C_2\sin\alpha\, k]\}.\end{aligned} \quad (94)$$

Hence the total force upon e_1 due to e_2 is

$$\mathbf{F} = e_1\left(\mathbf{E} + \frac{\mathbf{I}}{c}\mathbf{q}_1 \times \mathbf{H}\right) = \frac{e_1e_2\beta_2}{R^2A^3}\{-C_2\cos\alpha\, i + S_2 j - C_2\sin\alpha\, k\}$$
$$+ \frac{e_1e_2\beta_1\beta_2}{R^3A^3}\{[xS_1S_2 - a_1S_1^2S_2 + a_2S_1\cos\alpha + yS_1C_2\cos\alpha$$
$$- a_1S_1C_1C_2\cos\alpha]i + [-xC_1S_2 - a_1C_1S_1S_2 + a_2C_1\cos\alpha \qquad (95)$$
$$+ yC_1C_2\cos\alpha - a_1C_1^2C_2\cos\alpha]j + [zC_1C_2\cos\alpha - xC_1C_2\sin\alpha$$
$$+ yS_1C_2\sin\alpha + zS_1S_2 + a_2S_1\sin\alpha]k\}.$$

The Average Force.

In a crystal the arrangement of the axes of rotation of the atoms is such that the i and j components of the forces cancel out in summing up to get the force on a single atom due to the surrounding atoms. For this reason we will average only the z- or k-component of the above force. Also, since β is always a small quantity, the quantity A may be taken as unity without sensible error.

We obtain from the electric portion of (95) after averaging

$$\mathbf{F}_k = e^2\{P_1\sum_{P_2}(a\beta)(+Y\sin\alpha)\}r^{-3}k + e^2\{\sum_{P_1}a^2\sum_{P_2}(a\beta)[-\tfrac{5}{2}Y\sin\alpha$$
$$- \tfrac{1}{2}Y\sin\alpha\cos\alpha + 3Y^3\sin\alpha] + P_1\sum_{P_2}(a^3\beta)[-2Y\sin\alpha \qquad (96)$$
$$+ 3Y^3\sin\alpha + 3YZ^2\sin^3\alpha + 3X^2Y\sin\alpha\cos^2\alpha]\}r^{-5}k;$$

and from the magnetic portion

$$\mathbf{F}_k = e^2\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)\{+\tfrac{3}{4}X\sin\alpha + \tfrac{3}{2}Z\cos\alpha - \tfrac{15}{8}[+XZ^2\sin\alpha$$
$$+ X^2Z\cos\alpha + Y^2Z\cos\alpha]\}r^{-4}k. \qquad (97)$$

The series are here complete up to and including the r^{-5} term. It is seen that the electric part is in the odd and the magnetic part in the even powers, there being no terms in (97) in r^{-5}. We do not require any higher terms in this series than r^{-5} because the electrostatic terms, heretofore omitted, make their first appearance in the r^{-6} term. The above equations have the small factor β in the coefficients, and, when the r^{-6} and higher powers are added to the r^{-6} and higher electrostatic terms, the total of such terms does not differ from the electrostatic terms alone in an appreciable degree.

An examination of the result in (96) from the electric force shows that the quantity $Y\sin\alpha$ is a factor of every term. This is significant because the structure of cubic crystals is such that, because of this factor, the total force due to each and every plane of atoms in the crystal

vanishes. This is the case if the axes of the atoms in each plane are as shown in the paper on the diamond, figure 2, page 259, Phil. Mag., Aug., 1915. We conclude from this that the electric force contributes nothing to the total force upon any selected atom.

The magnetic portion (97) is the same as that formerly obtained from different equations with the exception of the numeric 15/8 which was 15/4 in the former result.[1] Following the plan on pages 264–5 of the paper on the diamond referred to, the forces on a selected atom are here recalculated in accordance with (97) for the β or r^{-4} terms, the r^{-6} or electrostatic terms remaining unchanged, being the same as in the papers referred to. That is, the only difference is in the numeric 15/8 instead of 15/4 in part of the r^{-4} terms only.

Plane Atoms

$$1 \quad J \quad \mathbf{F}_x = e^2\{-10.66666\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} + 142.22222\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}, \tag{98}$$

$$1 \ (20\text{–}25) \quad \mathbf{F}_x = e^2\{+0.07530\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} + 1.17036\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}, \tag{99}$$

$$1 \ (29\text{–}31) \quad \mathbf{F}_x = e^2\{-9.87654\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} + 117.6348\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}, \tag{100}$$

$$1 \ (32\text{–}34) \quad \mathbf{F}_x = e^2\{-0.08785\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} - 2.4363\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}, \tag{101}$$

$$3 \ (35\text{–}37) \quad \mathbf{F}_x = e^2\{-0.9750\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} - 5.757\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}, \tag{102}$$

$$\text{Total} \quad \mathbf{F}_x = e^2\{-21.5308\sum_{P_1}(a\beta)\sum_{P_2}(a\beta)l^{-4} + 252.83\sum_{P_1}a^2\sum_{P_2}a^2l^{-6}\}. \tag{103}$$

Since there is only one kind of atom in the diamond P_1 and P_2 are the same, and, equating to zero for equilibrium, we find

$$\left[\frac{\sum_P(a\beta)}{\sum_P a^2}\right]^2 = \frac{252.83}{21.53}l^{-2} = 11.73l^{-2}, \tag{104}$$

[1] Equation (25), p. 755, Phil. Mag., June, 1915.

whence

$$\frac{\sum_P(a\beta)}{\sum_P a^2} = 3.425l^{-1}. \tag{105}$$

Using the same values for the carbon atom as in the gravitational calculation (page 461), and denoting the radius of the outside ring of eight by a_0, which quantity we did not have to use before, we may obtain from (105) the value of a_0. We have from the atom

$$\sum_P(a\beta) = (8 + 4 \times .402^2)a_0\beta_0 = 8.6464a_0\beta_0 = .10376a_0, \tag{106}$$

$$\sum_P a^2 = 8.6464a_0^2. \tag{107}$$

Hence

$$\frac{\sum_P(a\beta)}{\sum_P a^2} = \frac{\beta_0}{a_0} = \frac{\omega}{c} = \frac{.012}{a_0} = 3.425l^{-1} \text{ by (105)}. \tag{108}$$

If we use the experimental value of l, the edge of the tetrahedron in diamond, namely 2.528×10^{-8} cm., we find

$$a_0 = \frac{.012}{3.425}\, l = .885 \times 10^{-10} \text{ cm.}, \tag{109}$$

or

$$l = 285.3a_0, \tag{110}$$

$$\sum_P(a\beta) = 9.18 \times 10^{-12}, \tag{111}$$

$$\sum_P a^2 = 6.77 \times 10^{-20} \quad \text{and} \quad \omega = 5.424 \times 10^{18}. \tag{112}$$

This result is in agreement with the theory of the atom which makes the radius of the orbit of the outside ring of electrons a small fraction, 1/285.3 of the distance between the centers of the atoms.

It seems incredible that the orbits of the electrons of the atoms in crystals can be any considerable fraction of the distance between the centers of the atoms, because of the interference between adjacent orbits that would result. The dimensions of the orbits demanded by the Rutherford-Bohr theory are incompatible with the above result, and they seem to be inadmissible from the standpoint of crystals alone irrespective of these results.

Compressibility and Bulk Modulus.

The proper theory of the forces interacting between the atoms in a crystal ought to make it possible to calculate the various mechanical properties of the crystal. This presents more difficulty if we begin

to deform the crystal so as to change the shape of the elementary space lattice formation, but, if we are content to imagine that a pressure is exerted upon all faces of the crystal so as to change its bulk only and leave the angles of the lattice undisturbed, then it is easy to calculate the change of volume produced by any applied pressure. The pressure per unit area divided by the change in volume per unit volume is known as the "bulk modulus," and it has been experimentally determined for a number of substances.

If ΔP denotes the pressure perpendicular to one of the principal planes of the diamond, the (111) plane, and if Δv is the change in volume of a centimeter cube of the substance due to this pressure, the bulk modulus is

$$M = \frac{\Delta P}{\Delta v}. \tag{113}$$

Before any external pressure is applied the crystal is in equilibrium, and the total force upon each atom zero. When the external pressure is applied on all sides so as not to change the formation, a contraction of the whole results until the external pressure is exactly balanced by the elastic forces brought into existence by a change in the size of the space lattice, and until the force upon each atom is again zero. Because of this lattice formation the whole force upon any one atom due to all the others, together with the external pressure, is completely expressed by equation (103), the total force being in the direction of the axis of rotation of each atom, whatever the absolute value of the edge of the elementary tetrahedron. Consider the atoms in a (111) plane. Such planes are each made up of equal numbers of atoms of four different orientations, the first having its axis perpendicular to this plane, and each of the other three groups being with axes of rotation making an angle $\cos^{-1}\frac{1}{3}$ with the normal to this plane. Since the total force on each atom is along its axis, the portion of the pressure due to one atom in each of the latter groups, resolved normal to the (111) plane, is $\frac{1}{3}$ of that due to the atom whose axis is normal to this plane. The resolved portion of the total force of one group of four atoms is, therefore, twice the force due to the perpendicular atom alone. If N is the number of such groups of atoms per square cm., we have

$$N = \frac{\sqrt{3}}{6}\frac{1}{l^2}. \tag{114}$$

The force on one atom per centimeter change in the linear dimensions is dF/dl, and for a small linear displacement Δx, the force per atom with

axis normal to the plane is $(dF/dl)\Delta x$. The force on one group is twice, and the force per sq. cm., ΔP, is $2N$ times this value. Hence

$$\Delta P = 2N\frac{dF}{dl}\Delta x = \frac{\sqrt{3}}{3l^2}\frac{dF}{dl}\Delta x. \tag{115}$$

The volume of the crystal considered is $v = x^3 = 1$ cu. cm. Hence $dv = 3x^2dx = 3dx$, and $\Delta v = 3\Delta x$. The bulk modulus (113) is, therefore,

$$M = \frac{\Delta P}{\Delta v} = \frac{\sqrt{3}}{9l^2}\frac{dF}{dl}. \tag{116}$$

The value of dF/dl may be obtained from (103) by differentiation, and we find, after substituting in the equilibrium value of l^{-2} from (104)

$$\frac{dF}{dl} = e^2 l^{-5}(\sum_P a\beta)^2(-43.18). \tag{117}$$

Using the values of these quantities above obtained, we obtain numerically

$$\frac{dF}{dl} = -.0802, \tag{118}$$

whence

$$M = 24.16 \times 10^{12}, \text{ if pressure is dynes per sq. cm.} \tag{119}$$

or

$$M = 24630 \times 10^6, \text{ if pressure is grams weight per sq. cm.} \tag{120}$$

The latter form is in the units usually given in the tables of experimental results. Some experimental determinations are given below for comparison, because the value for the diamond is not given in any of the tables. The figures given here are in the Smithsonian Physical Tables, p. 83.

Substance.	Bulk Modulus, Gms. per Sq. Cm.	Substance.	Bulk Modulus, Gms. per Sq. Cm.
Sylvine	138 × 10⁶	Tourmaline	9,140 × 10⁶
Rock salt	246 "	Lead	374 "
Quartz	387 "	Glass	405 "
Barite	535 "	Delta metal	1,012 "
Fluorspar	860 "	Brass	1,090 "
Pyrites	906 "	Copper	1,202 "
Beryl	1,384 "	Steel	1,518 "
Topaz	1,694 "		

The wide range of values for the crystals given in the table makes it impossible to tell the probable value for the diamond. The variation between sylvine and tourmaline is 66.2 times, or 6,620 per cent. The

value obtained theoretically for the diamond in (120) is between two and three times as large as the largest in the table, so that the order of magnitude of the calculated result is right. It may be remarked that Young's modulus for corundum has been measured, and found to be greater than that for any other substance yet tried. Although this is not the diamond, and it is not Young's modulus with which we are dealing, it does serve as a sort of indication that we may expect a large value of the bulk modulus for diamond. Subsequent calculations of this modulus for some of the substances which have been measured are to be desired.

SUMMARY AND REVIEW.

Having a conviction that some form of electromagnetic equations ought to be applicable to the individual electrons of which matter is composed, the first application should obviously be made to the steady states of matter at the absolute zero of temperature. In this condition it is supposed that the orbits of each electron making up the atoms are circles and that the motion in them is perfectly uniform. It is considered that any departure whatever from uniform circular motion will give rise to radiation or absorption of energy, but that this radiation is so small in the steady state of motion, if not exactly zero, that no change in the motion will result due to loss of energy through radiation in a period of time so great that it must entirely escape our observation.

The present form of electromagnetic theory already makes the radiation from a ring of electrons in the steady state very small. It has been supposed by some that the source of this energy is provided by a supposed secular diminution in the mass of the electron. In the words of Schott "the change of mass need only take place very slowly provided the velocity of the ring be small enough. In order that it should not contradict our experience as to the constancy of the properties of electrons and atoms, it is sufficient that $\dot{m}/m$ be less than, let us say 10^{-15}, or one thirty millionth part per annum, but of course it might be very much smaller."

In this paper a change in certain terms in the present form of the electromagnetic equations has been shown to be demanded, if they are to be applied to the atoms. The change suggested does not remove the equations from any connection with electromagnetic theory, and things point to a probable alteration either of the assumed equation of definition of the potentials ϕ and α, or of the assumed connection between charge and aether. If such a change is admitted to be required its effect upon the radiation terms is likely to be such as to introduce an additional factor of something like 10^{-31} into those terms that are already so small

as to cause Schott to make the remark above quoted. This would at once remove a very serious objection to the application of electromagnetic theory to the electrons in the steady states of matter. The source of this energy has been a serious stumbling block to the progress of the theory.

The single assumption is made that the electrons in the atoms of matter are moving at uniform velocities in circles at zero temperature, and the Lorentz form of electromagnetic equations is then applied to them. The fundamental problem to be solved is to find the average mechanical force which one such electron exerts upon another, when their orbits have the most general possible positions with respect to each other. This problem has been solved for the particular case of coaxial circles in a former communication, but is given above for the most general case. Obviously, until this problem was solved, we could not work with gross matter and crystal forms, because in such the axes of the atoms take many directions and the orbits are not coaxial circles.

The average force in the general case of two electrons in circular orbits is expressed as the sum of a series of terms of the inverse powers of the distance between the centers of the two orbits, the series beginning with the inverse first power and including all higher powers. When, however, this total force, which in general is not in the direction of the line joining the centers of the orbits, is resolved along this direction, the first power terms disappear, and the series begins with the inverse square of the distance between centers. If this distance is great enough, the first term of the series is the only term which is effective, so that at great distances the force resembles that of gravitation in this respect.

In all investigations prior to this the forces obtained between atoms have so depended upon the directions of the axes of rotation as to be an attraction or repulsion according to these directions, but these inverse square terms referred to always show an attraction. Although quantities involving the directions of the axes appear in these terms, they occur in such a way as to be ineffective and leave the total force always attractive. This is proved in the case of two crystals as well as other forms of matter including gases, liquids and solids.

The measurements by Mackenzie of the gravitational pull between lead masses, a lead mass and a crystal of calcspar, and finally between two crystals of calcspar seem to establish the fact that the gravitational force is independent of the directions of the axes, at least to the first order of approximation.

A disagreement between the calculated attraction and the existing gravitational attraction appears in the magnitude of the force only.

The calculated force is more than 10^{31} times greater than the existing force, and the difference is so large as to leave no doubt that something is fundamentally in error, and the future prospect of obtaining an all embracing electromagnetic theory depends, in part, upon a proper diagnosis of what is wrong. The hypotheses used are so few that the source of the error may be located, at least with some probability. These assumptions are two, that the electrons revolve in circular orbits in the atoms in their steady states, and, second, that the present form of the electromagnetic equations are applicable to these electrons.

The probability is that these equations *in their present form* do not apply to the electrons. This is the position already taken by a large number of physicists. But, the results indicate rather that there may be a fundamental error at some point in electromagnetic theory than that no such theory is applicable. And, if this can be found, we may still hope for a comprehensive theory, which at the same time embraces atomic phenomena and gravitation as well.

It is found that, by introducing a factor $\frac{1}{4}m\beta_1^2$ into the inverse square terms above mentioned, we then get the correct value of the gravitational pull to within about 1 per cent. It is surprising that a simple factor, having a direct physical significance connected with an electron, should be found that corrects the result. If there were no connection between the expression (54) and the true gravitational force, we would expect the conversion factor to be a pure numeric.

In tracing back to their source the terms from which (54) comes directly, it appears that this factor is introduced or required in those terms which come from the time differentiation of the potentials in distinction to the space differentiation, and that it is required not with every term of **E** and **H**, but in a peculiar manner, so that some terms are altered and others are not.

It is with regret that a solution for the mechanical force based upon new fundamental assumptions embracing this idea is not yet accomplished, and it seems best to publish these results as being of sufficient interest aside from any attempt to build upon a slightly different electromagnetic foundation.

Four properties have been examined by means of modified equations,

1. The equlibrium speed of rings of electrons.

2. The form of the force-equation applicable to atoms at close range, as in the formation of molecules, with application to the diamond as one type of molecule.

3. The elastic forces brought into existence by the compression of the diamond so as not to alter the angles of the space lattice formation, the calculation of the bulk-modulus.

4. The attraction between two diamonds, each 10^{-18} cubic cm. at one cm. apart, oriented in all possible ways with respect to each other.

It is shown that, for rings of electrons, the modified equation gives the same result to a very close degree of approximation as did the original form.

In the fourth case the modified equation gives the gravitational force within about 1 per cent., while the original form gives a force more than 10^{31} times greater than the actual force.

In the third case we obtain the proper order for the value of the bulk-modulus. This result alone goes to show that we are dealing with interatomic forces of the right order of magnitude, and renders it probable that the inverse fourth power terms are the first to be effective in the force-series at the distances concerned between atoms in a crystal. These results lead to a determination of the radius of the orbit of the outside ring in a carbon atom, and confirm former results as to its small dimensions $.885 \times 10^{-10}$ cm.

YONKERS, N. Y.

AN AIR-BLAST RECTIFIER OF HIGH TENSION ALTERNATING CURRENTS.

BY E. R. WOLCOTT AND C. J. ERICKSON.

THAT a partial rectification of a high-tension alternating current occurs on the passage of a discharge from a point to a plate, has been known for many years. However, difficulty has been encountered in attempting to utilize this rectification, particularly if any considerable current flows, as there then results a heavy arc which rectifies but little.

An examination of the effect of a blast of air on this discharge showed that complete rectification and smooth operation could be obtained by means of a current of air flowing from the point to the plate.[1] A method of accomplishing this is illustrated diagrammatically in Fig. 1.

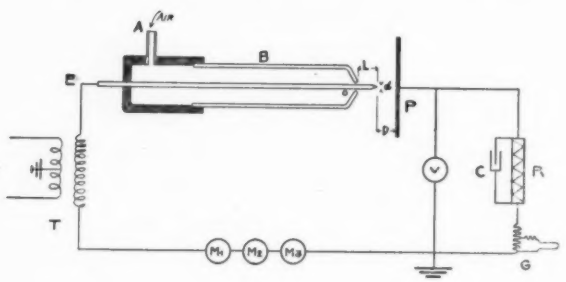

Fig. 1.

A pointed metallic electrode E is mounted coaxially within the tube B, which is restricted at one end so that a gas entering at A is blown through the opening O toward the plate P. Rectification occurs when the point is some distance within the tube, at the opening of the tube, or extending some distance outside of the tube as shown. In the first two cases, the tube B must be made of some insulating material, as of glass; in the last case it may be of metal.[2]

Mounted for use, this rectifier was connected to one terminal of the secondary of a 2 K.W., 40,000-volt transformer T, the other terminal being grounded. The positive current flows from the point to the plate,

[1] E. R. Wolcott and F. Rieber, U. S. Patent No. 1,188,597.

[2] S. D. Browning and C. J. Erickson, thesis University of California, under direction of Prof. Harmon F. Fischer.

and no discharge results in the opposite direction when the distance between the point and plate, that between the point and air outlet, the air pressure and the voltage are suitably adjusted.

Connected between the other terminal of the transformer and the ground were three milliammeters: M_1 was of the electro-dynamometer type; M_2 was a direct-current instrument of the D'Arsonval permanent field type; M_3 was a hot-wire instrument.

The secondary voltage from the plate P to ground was measured by an electrostatic voltmeter. A General Electric oscillograph was mounted directly in the high-tension circuit between the water resistance R and the ground.

The air condenser C consisted of eleven plates 8 by 3 feet (2.44 by 0.91 meters) and 8 inches (20.3 cm.) apart. Its capacity was 0.00095 microfarads and it was connected in parallel with the water resistance when desired.[1]

The diameter of the plate P should be large enough to prevent a discharge from the point to its edges. In the arrangement here described, its diameter was 4 inches (10.16 cm.). The diameter of hole O in the tube B was 0.157 inches (4 millimeters). d, the diameter of the metallic electrode E, was 0.0625 inches (1.57 millimeters). L, the distance between the opening O and the point of the electrode E, was 0.375 inches (9.54 cm.). D, the distance between the plate and the point, was 0.68 inches (1.76 cm.). The frequency of the alternating current was 60 cycles per second.

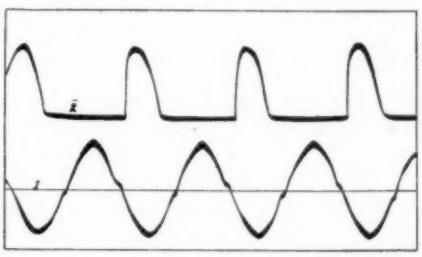

Fig. 2.

With no air flowing out of the opening O of the tube B, curve 1 was obtained at 25 kilovolts. The discharge in this case was an alternating current arc. With air flowing out of the opening O, under a pressure of

[1] The curves shown in Figs. 2 and 3 were obtained with no condenser, those shown in Figs. 4 and 5 were obtained with the condenser connected in parallel with the resistance. One vibrator was connected, in series with a suitable resistance, in the condenser circuit to show the condenser load current. The discharge of the condenser through the parallel resistance is indicated by the drop of the condenser load current curve below the zero line.

2 inches (5.08 cm.) of mercury, curve 2 was obtained. These two curves were not obtained simultaneously so the phase relations as shown are not correct. The direct current reading was 12.3 milliamperes, and the voltage 14.5 kilovolts.

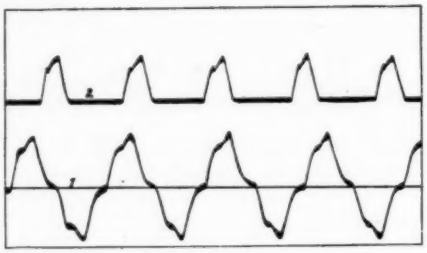

Fig. 3.

This curve differs from Fig. 2 in that a different alternating current generator was used. It will be noticed that the half wave passed by the discharge follows, in general, the wave form of the alternator; but, since a certain voltage is required to start the discharge, it does not include the whole of the half wave. The rise of the rectified wave is more abrupt than that of the original wave.

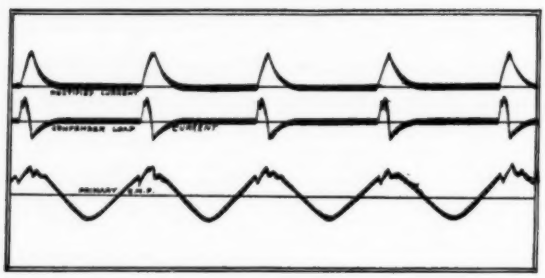

Fig. 4.

These curves were not taken under exactly identical conditions of Figs. 2 and 3 but all three were taken simultaneously and are in phase. The primary electromotive force (E.M.F.) was 241 volts; the secondary voltage was 15,250 volts; the rectified current was 25 milliamperes.

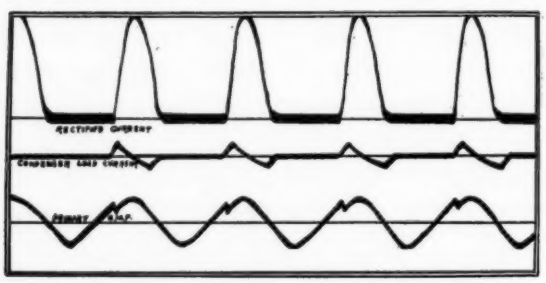

Fig. 5.

These curves were taken under the same conditions as those of oscillogram Fig. 4 except the load was heavier, that is, 104 milliamperes. The sudden drop in primary voltage, at the moment of the application of the load, is plainly evident.

The discharge, when rectification results, is white, rhythmic and snappy, as distinguished from the colored arc which results when no air is flowing. The latter rectifies to a small extent, but in the opposite direction.

Difficulties are encountered trying to operate at too low voltages, say only a few thousand volts. A certain voltage is required to jump the air gap. Likewise there is a drop in voltage across the rectifier in actual operation. Satisfactory rectification has been obtained up to 350 kilovolts, and also at frequencies up to 500 cycles per second.[1]

With the point inside of a glass tube B, the discharge is more snappy, but the tendency to surge is greater. In this case aluminum electrodes seemed preferable, a larger plate P was necessary, and a greater air pressure required.

With the point outside of the tube, satisfactory results were obtained with iron and brass electrodes.

As the gas pressure increases from zero during the discharge, the rectification increases to a maximum, the value of which depends upon the setting of the electrodes and the diameter of the gas outlet.

A further increase of gas pressure beyond this value which gives maximum rectification, produces irregular operation.

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Los Angeles, California.

[1] K. V. Laird, Fog Dispersion Tests, Panama-Pacific International Exposition, under the direction of Dr. F. G. Cottrell.

THE THEORY OF ELECTROMAGNETIC MASS OF THE PARSON MAGNETON AND OTHER NON-SPHERICAL SYSTEMS.

BY DAVID L. WEBSTER.

INTRODUCTION.

IN the remarkable theory of atomic structure recently proposed by Parson[1] it is assumed that the electron is not spherical, as previously supposed, but a very thin ring, about 1.5×10^{-9} cm. in radius, on which the negative charge revolves at a very high velocity, of the order of that of light. This gives the electron, or magneton, as he calls it, a combination of electrostatic and steady magnetic properties, the steadiness of these magnetic properties being essential to the theory. Their detection by direct experiment with molar magnetic fields was shown by Parson to be very difficult, owing to the fact that the magneton is electrostatically single but magnetically a neutral doublet. Evidence of this sort has, however, been obtained by Grondahl[2] for free electrons within an iron wire, that seems to agree satisfactorily with calculated effects. It might be supposed that evidence would be obtained also from the fact that the inequality of the magnetic energies of a magneton moving along its axis or perpendicular to it would give a tendency for high speed cathode rays to orient themselves in one plane and produce magnetic and other effects. Or one might expect the same influence in a non-rotating monatomic atom such as one of mercury vapor to produce polarization in the light emitted by it. But as Ehrenfest[3] and others have pointed out, such effects would give direct evidence of absolute motion, and therefore are prohibited by the principle of relativity.

Notwithstanding this, the electromagnetic mass and momentum, m_e and M_e, and the magnetic and electrostatic energies T_e and W_e, are very different for motions with the same velocity in different directions in systems not having spherical symmetry, and one may readily prove that $T_e + W_e$ is not constant although $T_e - W_e$ is so. Therefore in such cases we must have some other forms of mass, momentum and energy, due to some other cause than the electromagnetic field. Much as we

[1] "A Magneton Theory of the Structure of the Atom," Smithsonian Miscellaneous Collections, Nov., 1915.

[2] Amer. Phys. Soc., Dec., 1916, PHYS. REV. [2], 9, 1917.

[3] Ann. d. Phys., 23, 204–5, 1907.

may hesitate to speculate on what is inside as small a thing as the magneton, there seems to be no other cause for this extra mass than the changes occurring on acceleration in the internal forces that hold the magneton together. This compels us to test the possibilities of various assumptions about them in search of a possible explanation of this "internal mass." One such explanation is that which I outlined in a discussion before the American Physical Society in December, 1916, and more fully in February, 1917, and which is described in an abstract in the PHYSICAL REVIEW of 1917. The purpose of the present paper is to give an actual proof of the assertions made there.

The analysis whose results are outlined in this paragraph is designed primarily for the magneton theory; but it is equally necessary for and applicable to any system of static charges and steady currents (that is, if ρ and ρv are independent of t). Let each pair of charge or current elements be imagined connected by a string or rod that will balance their forces on each other when the system is at rest. When the system is accelerated, the non-radiated part of the electromagnetic field from element A upon element B will appear as though A was not where it actually is, but where it would be if it had not been accelerated since the radiation left it which is now arriving at B. If now the internal force on B due to A is governed by this fictitious position of A rather than its real position, but is of the same strength as though the fictitious distance to A were equal to its real distance, then these internal forces will give a mass effect varying with the direction of acceleration in exactly the required way.

This is not the only system of internal forces that will accomplish the result, and for that reason the internal mass may differ from the mass given by this system by any constant amount. A result of this is that the relation between the mass of the classical electron (if it exists) and its radius is not determined so unambiguously as it is supposed to be, and would change by 50 per cent. if a set of forces of this type were substituted for the hydrostatic tension postulated by Lorentz. If the forces in the magneton are of the type suggested here, it appears that the internal forces balancing magnetic actions will give a negative mass always exactly balancing the positive magnetic mass, and that the mass of the magneton is determined solely by its electrostatic properties and corresponding internal forces. It should, however, be remembered that the internal forces are not definitely known, and therefore that the total mass of any electrical system is really uncertain, though the necessity for some internal mass of any non-spherical system is a direct consequence of the principle of relativity.

In any case, there must be an internal mass, m_i, and momentum, M_i, given presumably by the forces required to hold the system in shape against the electrical forces and these must satisfy the equations:

$$m_i = m - m_e, \quad \text{and} \quad M_i = M - M_e. \tag{1}$$

Thus to find the relation between the mass of the magneton and its charge, current, and dimensions, we must investigate these internal forces in detail.

THE MOMENTUM AND ENERGY OF A MOVING ELECTROSTATIC SYSTEM.

Before undertaking the treatment of the more complicated case of a magneton, let us obtain some idea of how all this may happen by considering a simpler hypothetical case of two electrons of the classical type, of charge $+ e$ and radius R held at a distance a which is constant except for Lorentz-Fitzgerald contraction.

We shall denote the electric and magnetic vectors measured in the standard electrostatic and magnetic systems by $\boldsymbol{e}$ and $\boldsymbol{b}$ respectively, these letters being used rather than Lorentz's $\boldsymbol{d}$ and $\boldsymbol{h}$ because their averages over "physically infinitesimal" volumes are the $\boldsymbol{E}$ and $\boldsymbol{B}$ of ordinary electromagnetics, rather than $\boldsymbol{D}$ and $\boldsymbol{H}$. The "time" as measured by the distance, ct, that light has travelled since $t = 0$ will be denoted by l, and the units of momentum and mass will be those required by the use of l, that is, the ordinary ones divided by c and c^2 respectively. The value of $\boldsymbol{v}/c$ for any bit of electricity will be denoted by $\boldsymbol{u}$, and that of the system as a whole by $\boldsymbol{\beta}$. Vectors will be denoted by Clarendon type, and their components when treated as scalars will be in italics.

For the general case of any system we have

$$\left.\begin{aligned} \boldsymbol{M}_e &= \frac{1}{4\pi}\int_\infty \boldsymbol{e} \times \boldsymbol{b}\, d\tau, \\ T_e &= \frac{1}{8\pi}\int_\infty \boldsymbol{b}^2 d\tau, \quad W_e = \frac{1}{8\pi}\int_\infty \boldsymbol{e}^2 d\tau, \end{aligned}\right\} \tag{2}$$

where $\boldsymbol{e} \times \boldsymbol{b}$ denotes the vector product of $\boldsymbol{e}$ and $\boldsymbol{b}$.

The vectors $\boldsymbol{e}$ and $\boldsymbol{b}$ may be found by the equations

$$\left.\begin{aligned} \boldsymbol{e} &= -\nabla\varphi - \dot{\boldsymbol{a}}, \quad \boldsymbol{b} = \nabla \times \boldsymbol{a}, \\ \nabla^2\varphi - \ddot{\varphi} &= -4\pi\rho, \quad \nabla^2\boldsymbol{a} - \ddot{\boldsymbol{a}} = -4\pi\rho\boldsymbol{u}, \quad \nabla\cdot\boldsymbol{a} + \ddot{\varphi} = 0, \end{aligned}\right\} \tag{3}$$

where ∇ is the vector operator $(\boldsymbol{k}_x D_x + \boldsymbol{k}_y D_y + \boldsymbol{k}_z D_z)$, $\boldsymbol{k}_x$, $\boldsymbol{k}_y$ and $\boldsymbol{k}_z$ being unit vectors. Thus $\nabla\varphi$ is the gradient of the scalar potential φ, and $\nabla \times \boldsymbol{a}$ and $\nabla\cdot\boldsymbol{a}$ the curl and divergence, respectively, of the vector potential $\boldsymbol{a}$, the dot between two vectors denoting the scalar product.

$\dot{\varphi}$ denotes $D_t\varphi$. These equations, in a different notation, and many that we shall derive from them, are given in Lorentz's "Theory of Electrons."

For the special case of the two electrons, we may use Lorentz's method of dealing with a static system in motion. Taking x in the direction of $\boldsymbol{\beta}$, we have $\boldsymbol{u} \equiv \boldsymbol{\beta}$, and $\dot{\varphi} = -\beta D_x\varphi$, $\ddot{\varphi} = \beta^2 D_x^2\varphi$, etc., so that if we let $k = (1-\beta^2)^{-1/2}$ and $x' = k(x - \beta l)$ we may put (3) in the form

$$\nabla'^2\varphi = -4\pi\rho, \quad \nabla'^2\boldsymbol{a} = -4\pi\rho\boldsymbol{\beta}, \quad \boldsymbol{a} = \varphi\boldsymbol{\beta}, \tag{4}$$

or by introducing $\rho' = k^{-1}\rho$ and $\varphi' = k^{-1}\varphi$, $\boldsymbol{a}' = k^{-1}\boldsymbol{a}$ we have

$$\nabla'^2\varphi' = 4\pi\rho', \quad \boldsymbol{a}' = \varphi'\boldsymbol{\beta} \tag{5}$$

with the values of ρ' in the x', y, z system the same as ρ at corresponding points in an x, y, z system when everything is stationary. This, of course, is merely a special case of the relativity transformations.

From these equations, by steps given in full by Lorentz, note 14, we may prove

$$W_e = \frac{1}{8\pi}\int_\infty \{k^{-1}(D_{x'}\varphi')^2 + k[(D_y\varphi')^2 + (D_z\varphi')^2]\}d\tau', \tag{6}$$

$$T_e = \frac{1}{8\pi}\beta^2 k\int_\infty \{(D_y\varphi')^2 + (D_z\varphi')^2\}d\tau', \tag{7}$$

$$\left.\begin{aligned} M_{ex} &= \frac{1}{4\pi}\beta k\int_\infty \{(D_y\varphi')^2 + (D_z\varphi')^2\}d\tau', \\ M_{ey} &= \frac{1}{4\pi}\beta\int_\infty D_{x'}\varphi' D_y\varphi' d\tau', \\ M_{ez} &= \frac{1}{4\pi}\beta\int_\infty D_{x'}\varphi' D_z\varphi' d\tau'. \end{aligned}\right\} \tag{8}$$

Thus,

$$\left.\begin{aligned} T_e &= \tfrac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{M}_e, \\ T_e - W_e &= -k^{-1}W_{eo}, \end{aligned}\right\} \tag{9}$$

where W_{eo} is the electrostatic energy of the system at rest, so that $T_e - W_e$ is the same for all directions. (For the general case one may readily prove from the relativity transformations that

$$T_e - W_e = k^{-1}(T_{eo} - W_{eo}).)$$

To evaluate M_e, we might use equations (8) but shall not do so, as there is a shorter way. For, by equation (2), the result must be the same as if we assume all electric and magnetic energy to be transported as indicated by Poynting's vector $\boldsymbol{e} \times \boldsymbol{b}$ and to have momentum equal, in the units of this paper, to its value as energy times the velocity with

which it would have to move to give the required transfer. This statement emphasizes the pragmatic value of this assumption, and is independent of what may or may not be said about actual truth of the assumption. Here we shall limit ourselves to a very slow motion, so that the total magnetic energy, being less than β^2 times the electrostatic, may be neglected.

The assumed flow of energy must be fast enough to accomplish three things: first, it must transport the whole electrostatic energy bodily with the system at a velocity $\boldsymbol{\beta}$; second, since energy is developed in the field at the back of each electron by the internal stresses pulling the electricity along against the retarding electric force, it must be transferred forward around the electron to be given up to the internal stresses again; and third, when the direction of motion is that of a, a similar transfer is necessitated by the force holding them together against their repulsion.

The first part gives $\frac{1}{2}(e^2/R)\boldsymbol{\beta}$ for each electron and $(e^2/a)\boldsymbol{\beta}$ for their mutual energy. For the second part we may notice that as the back of the electron sweeps over any element of volume, the work done by the internal stress equals the product of the volume of the element by the tension stress holding the surface of the electron from expanding. If dS is the element of surface, and x its distance forward of the center the energy given to the electromagnetic field per unit time is

$$\frac{1}{2}\frac{e}{R^2}\cdot e\frac{dS}{4\pi R^2}\cdot\frac{-x}{R}\beta$$

and the distance this must be carried forward by the field is $2x$, making the contribution of this element to $\boldsymbol{M}_e$ equal to $(e^2\boldsymbol{\beta}x^2/4\pi R^5)dS$. Now $(x/R)dS$ is the projection of dS on the y, z plane, and therefore $(2x^2/R)dS$ is the volume in a cylinder parallel to the x axis on this element. Integrating, therefore, we have for the total contribution of this sort to $\boldsymbol{M}_e$, $\frac{1}{6}(e^2/R)\boldsymbol{\beta}$, making the total mass of one electron the same as one obtained by direct integration.

The last term may likewise be found to be $(e^2/a)\beta$ in the case of motion parallel to the line of centers, and zero for motion perpendicular to it.

Thus the total electromagnetic momentum is $\frac{4}{3}(e^2/R)\boldsymbol{\beta}$ for both electrons separately, and $(2e^2/a)\boldsymbol{\beta}$ or $(1e^2/a)\boldsymbol{\beta}$ for the mutual momentum, according to the direction of motion. The total magnetic energy, T_e, being $\frac{1}{2}\boldsymbol{\beta}\cdot\boldsymbol{M}_e$ undergoes similar changes with direction. Notwithstanding this, the system must be free to turn in any position without external forces or gain or loss of velocity. The next problem is therefore to find the source of momentum and energy that makes up for these changes in $\boldsymbol{M}_e$ and T_e.

To determine this, let us examine the mutual electromagnetic mass from another viewpoint. In the case of acceleration along the line of centers there is a retarding action of the ether due to the delay in the changes of the electrostatic force by the time a required for propagation from one electron to the other. This makes a difference between the forces acting on them, equal to

$$2\frac{d}{da}\left(\frac{e^2}{a^2}\right)\cdot\tfrac{1}{2}\dot{u}a^2 = -\frac{2e^2}{a}\dot{u},$$

which, acting for the time $\beta/\dot{u}$ required to gain the velocity β, will produce an impulse

$$-\frac{2e^2}{a}\beta,$$

which must be overcome by the external forces. The mutual electromagnetic mass is therefore the same in this case, that we found by the other analysis.

For acceleration across the line of centers we have a similar effect due in this case to a change of direction and giving a forward component of force on each one equal to

$$\frac{e^2}{a^2}\cdot\frac{\tfrac{1}{2}\dot{u}a^2}{a} = \tfrac{1}{2}\frac{e^2}{a}\dot{u}.$$

Also the electric vector radiated from each one, which was zero in the other case, gives a component on the other electron equal to $-(e^2/a)\dot{u}$. Thus, by a calculation exactly like that of the other case, we have a mutual electromagnetic mass of $1e^2/a$, rather than $2e^2/a$. The most plausible way to satisfy equations (1) is to account for the constancy of the total mass by some effect of the forces that hold the electrons together.

In rapid steady motion along the line of centers the distance a is contracted to ak^{-1}, while the electrostatic repulsion remains the same as it is at rest; and therefore the other force, of attraction, may plausibly be assumed independent of this distance. Hence in longitudinal acceleration it may be assumed the same for both in spite of the difference of the electrostatic forces, and we may say

$$m_{li} = 0, \quad m = m_{le} = \frac{4}{3}\frac{e^2}{R} + 2\frac{e^2}{a}. \tag{10}$$

Therefore, since

$$m_{te} = \frac{4}{3}\frac{e^2}{R} + \frac{e^2}{a} \tag{11}$$

we must have

$$m_{ti} = \frac{e^2}{a}. \tag{12}$$

This is accounted for if we assume that the internal attraction is propagated with the velocity c in such a way that for each one at a time l it is always directed to the position where the other one would be if it had kept, since the time $l - a$, the velocity it had then. This position differs from the real one by the distance $\frac{1}{2}\dot{u}a^2$, and since the magnitude of the force is e^2/a^2, this effect would give exactly the required drag, $-(e^2/a)\dot{u}$, on the system. That means that this force is propagated with the velocity of radiation but without the perpendicular components characteristic of radiation.

While this assumption is of no interest for its own sake, since the case is a purely hypothetical one, it is of interest in showing the type of assumptions that may be made about the internal forces of the magneton or any other system to account for the uniformity of its mass in all directions which is demanded by equation (1) in the name of the principle of relativity and the experiments on cathode and β rays. Moreover it points toward the conclusion that every set of forces which carry energy with them must also give rise to an amount of mass in any direction equivalent to the energy carried by this set of forces when moving in that direction. Although this mass is not explained on an electromagnetic basis, it is correlated with other forces very much as electromagnetic mass is correlated with electromagnetic forces, as a result of the propagation of the forces by the ether with the velocity of light, though the assumed laws of propagation are distinctly different in the two cases.

An interesting consequence of the necessity for this "internal" mass, as we may call it, is a certain indefiniteness in the total mass of a classical electron. For if the internal stress is really a simple hydrostatic tension, as Poincaré and Lorentz have supposed, then there is no internal mass at all. But if the stress is a system of bonds between opposite elements of the charge, like the bond between the two electrons that we have assumed in the hypothetical case, then the internal energy is three times that of the hydrostatic tension and there is along with it an internal mass of $\frac{1}{3}e^2/R$, making the whole mass equal to e^2/R, instead of $\frac{2}{3}$ that amount.

This hypothesis in the case of a single classical electron would be a complication that is unjustified except perhaps by considerations of stability of shape of the electron.[1] Nevertheless the existence of the mass of this type is certainly necessary in cases such as the magneton,

[1] See Lorentz, l.c., Chapter 5, pp. 214–5. Lorentz notes here the probability that the internal stress in the electron may be more than a simple hydraulic tension, which would give a stable volume but not a stable shape.

in which the internal stresses cannot be purely hydrostatic.[1] In such cases therefore we must always recognize the dependence of the total mass on the nature of the internal stresses, and the resulting uncertainty in any inferences from the mass of the system as to the dimensions.

A General Theorem on "Internal" Masses.

Let us consider now the more general case of any system in which the charge densities and current densities at all points are constant, except for the changes required by relativity. This includes the case of the magneton or any atom that is not undergoing any change. In such cases we have, so long as the system is at rest, no displacement currents or time changes of ρ and $\rho\boldsymbol{u}$.

There are many possible sets of internal forces that will keep such a system in equilibrium when it undergoes the changes demanded by relativity on acquiring a high velocity, and it may be that there are many of them that would give a constant mass with some reasonable laws governing their behavior when the system is accelerated. The purpose of this paper, however, is not to investigate how many there are, or what laws each one would require, but rather in addition to pointing out the constancy of the mass of a system to find some plausible explanation of the non-electromagnetic part.

Therefore let us prove the following theorem: Let the internal forces of the system whose densities of charge and current do not vary with time be two sets of pairs of forces between each pair of volume elements, one of which sets exactly balances the electrostatic forces between these elements when the system is permanently at rest while the other set corresponds in the same way to the magnetic ones; let these forces be independent of the "apparent distances" between the elements; but vary with apparent direction exactly like the electromagnetic forces to which they correspond; when the system is accelerated, let each force of such a pair act at each instant as though the other element had kept its velocity constant since the instant when radiation would have had to start from it to reach the element in question at the present time; then the resultant of all these forces will combine with the electromagnetic field to produce a mass which is the same in all directions when the system is accelerated from rest.

As a corollary to this we may prove that under these conditions the mass due to the magnetic field is always exactly balanced by a negative mass due to the corresponding internal forces.

[1] The impossibility of assuming in the magneton such a distribution of charge and current as to give no internal forces seems to be proved by the excessive thinness of the ring found.

First, to prove that the effects of the electric and magnetic fields are independent we may find the electromagnetic momentum of such a system for an infinitesimal velocity β, by using the relativity transformations with all terms involving β^2 neglected. Thus, letting primes denote quantities as measured by an observer moving with the system, and therefore equal to the values the corresponding real quantities would have if the system were at rest, we have

$$\begin{cases} \boldsymbol{e}' = \boldsymbol{e} + \beta \times \boldsymbol{b}, & \boldsymbol{b}' = \boldsymbol{b} - \beta \times \boldsymbol{e}, \\ \boldsymbol{u}' = \dfrac{\boldsymbol{u} - \beta}{1 - \beta \cdot \boldsymbol{u}}, & \rho' = \rho(1 - \beta \cdot \boldsymbol{u}). \end{cases}$$

Since to the moving observer the system appears steady we have

$$\boldsymbol{M}_e' = \frac{1}{4\pi} \int_\infty \boldsymbol{e}' \times \boldsymbol{b}' d\tau' = 0, \tag{13}$$

for any system as symmetrical as the magneton.
But in any case,

$$\begin{aligned} \boldsymbol{M}_e' &= \frac{1}{4\pi} \int_\infty [\boldsymbol{e} \times \boldsymbol{b} + (\beta \times \boldsymbol{b}) \times \boldsymbol{b} - \boldsymbol{e} \times (\beta \times \boldsymbol{e})] d\tau' \\ &= \boldsymbol{M}_e - \frac{\beta}{4\pi} \int_\infty [\boldsymbol{b}_t^2 + \boldsymbol{e}_t^2] d\tau' + \frac{1}{4\pi} \int_\infty [\boldsymbol{k}_{b_t} \beta b_t b_l + \boldsymbol{k}_{e_t} \beta e_t e_l] d\tau', \end{aligned} \tag{14}$$

where the subscripts t and l denote the directions perpendicular and parallel to β, respectively.

But since $\boldsymbol{M}_e - \boldsymbol{M}_e'$ represents the electromagnetic momentum due to the velocity β, this equation shows that the steady electric and magnetic fields have no mutual mass.[1] This might be guessed from the fact that they have no mutual energy but does not follow from it without proof.

The electric mass alone is therefore the same as if no currents existed, and the magnetic mass the same as if there were no electrostatic charges.[2]

[1] Such a theorem has been proved previously by Comstock, Phil. Mag. (6), 15, pp. 1–21, Jan., 1908, but the proof given here, based on relativity, seems worthy of attention since it is considerably shorter and more direct and its range of application is different from that of Comstock's theorem.

[2] Equation 14 enables us to evaluate approximately the ratios of the masses of the electrostatic and magnetic fields of the magneton for each of the principal directions. As we shall prove below, the ratio of the radius of the cross section of the ring to that of the ring itself is extremely minute, and most of the energy and momentum of the field are concentrated very closely around the ring. This enables us to use the formulas for a straight wire as a first approximation. In this way one may readily prove that the momentum of the static charge is approximately the velocity of the magneton times its electrostatic energy in the case of motion along the axis, or times 3/2 the electrostatic energy for motion perpendicular to the axis. Similarly the steady current has a momentum approximately equal to the velocity of the system times the magnetic energy when moving along the axis, or 3/2 of this value when moving perpendicularly to it.

To prove the theorem we now have to find first some system of internal forces that will balance all those of electrostatic origin and at the same time obey the other conditions. Then we may do the same for the magnetic forces.

Such a system for the electrostatic forces must give a force of attraction of magnitude

$$\frac{\rho\rho' d\tau d\tau'}{r^2}$$

between any two elements $d\tau$ and $d\tau^1$ whose distance is r when the system is permanently at rest; and the direction of each of them must be toward the position the other element would occupy if its velocity had not changed for a time r before the instant in question. This will put these elements in the condition of the two electrons considered above, and thus make the total direct and indirect electric mass of every pair of elements constant.

To find the magnetic mass, we may treat the system as if every element were electrostatically neutral, but had the same current density that it really has. $\boldsymbol{b}$ may then be found by equation (4), where

$$\boldsymbol{a} = \int_\infty \frac{[\boldsymbol{q}]d\tau}{r}, \tag{15}$$

the brackets indicating that the value of $\boldsymbol{q}$, or $\rho\boldsymbol{u}$ (the current density vector), for the element $d\tau$ is a "retarded" one, taken for a time an amount r before the time in question.

When the system is permanently at rest, $\boldsymbol{q}$ does not change with time, and $[\boldsymbol{q}] = \boldsymbol{q}$, so that the retarded values present no complications. But if the system is in motion with a low velocity $\boldsymbol{\beta}$ we must take account of the fact that the distance to the retarded position of each element is not that of the present position, and that the effective volume also is different. By exactly the reasoning given by Lorentz for the case of an electron in motion, we may show that this gives

$$\boldsymbol{a} = \int_\infty \frac{\boldsymbol{q}d\tau}{[r(1 - \beta_r)]}, \tag{16}$$

where β_r is the component of $\boldsymbol{\beta}$ in the direction from $d\tau$ to the point where $\boldsymbol{a}$ is to be found.

To find $\boldsymbol{b}$, or $\nabla \times \boldsymbol{a}$, for cases where β_r is small we may remember that if $\boldsymbol{\beta}$ is constant, $[r(1 - \beta_r)] = r +$ (terms involving β^2), so that $\boldsymbol{b}$ is the same as when the system is at rest except for such terms. In other cases we must have $\boldsymbol{b}$ equal to this part plus terms involving $\dot{\boldsymbol{\beta}}$; and since the static part produces no resultant force on the whole system, the part

involving $\dot{\boldsymbol{\beta}}$ must be held responsible for the magnetic mass. We shall neglect all terms involving $\boldsymbol{\beta}^2$ and $\boldsymbol{\beta}\cdot\dot{\boldsymbol{\beta}}$.

To find $d\boldsymbol{b}$, the part of $\boldsymbol{b}$ due to the element $d\tau$, we must compute

$$\nabla \times (d\boldsymbol{a}) = \nabla \times \frac{\boldsymbol{q}d\tau}{[r(1-\beta_r)]}, \tag{17}$$

where $\boldsymbol{q}$ has the proper value for the element $d\tau$ and the symbol ∇ refers to differentiation with respect to the position of the other end of r. Now $[r] = r + [r\beta_r] + \frac{1}{2}[r^2\dot{\beta}_r] = r + r\beta_r - \frac{1}{2}r^2\dot{\beta}_r$, and $[1-\beta_r] = 1 - \beta_r + r\dot{\beta}_r$, so that the denominator of $d\boldsymbol{a}$ is $r(1+\frac{1}{2}r\beta_r)$.

To find $\nabla \times (d\boldsymbol{a})$ let us use polar coördinates with the present position of the element as origin and direction of $\boldsymbol{q}$ as the pole, and with φ measured from the plane that includes the directions of $\dot{\boldsymbol{\beta}}$ and $\boldsymbol{q}$. With these coördinates

$$\begin{aligned}\nabla \times (d\boldsymbol{a}) = \boldsymbol{k}_r &\left\{ \frac{1}{r\sin\theta} D_\theta(\sin\theta\, da_\phi) - \frac{1}{r\sin\theta} D_\phi(da_\theta) \right\} \\ &+ \boldsymbol{k}_\theta \left\{ \frac{1}{r\sin\theta} D_\phi(da_r) - \frac{1}{r} D_r(r\,da_\phi) \right\} \\ &+ \boldsymbol{k}_\phi \left\{ \frac{1}{r} D_r(r\,da_\theta) - \frac{1}{r} D_\theta(da_r) \right\},\end{aligned} \tag{18}$$

where

$$d\boldsymbol{a} = \frac{\boldsymbol{q}d\tau}{r(1+\frac{1}{2}r\dot{\beta}_r)} = \frac{\boldsymbol{q}d\tau}{r} - \tfrac{1}{2}\boldsymbol{q}\dot{\beta}_r d\tau. \tag{19}$$

The part $\boldsymbol{q}d\tau/r$ gives the value $\boldsymbol{b}$ would have if $\dot{\boldsymbol{\beta}} = 0$, that is, if the system were unaccelerated. This part is known to give no resultant force on the system, so that the part to be considered is

$$d\boldsymbol{a}' = -\tfrac{1}{2}\boldsymbol{q}\dot{\beta}_r d\tau. \tag{20}$$

Since the direction of $d\boldsymbol{a}'$ is always that of $\boldsymbol{q}$,

$$da_\phi' \equiv 0, \quad da_r' = -\tfrac{1}{2}q\dot{\beta}_r\cos\theta\, d\tau, \quad da_\theta' = +\tfrac{1}{2}q\dot{\beta}_r\sin\theta\, d\tau.$$

Let us now subdivide $d\boldsymbol{a}'$ into two parts: $d\boldsymbol{a}''$, due to $\dot{\boldsymbol{\beta}}''$ the component of $\dot{\boldsymbol{\beta}}$ parallel to $\boldsymbol{q}$; and $d\boldsymbol{a}'''$, due to $\dot{\boldsymbol{\beta}}'''$, the rest of $\dot{\boldsymbol{\beta}}$. Thus we have

$$da_\phi'' = 0. \quad da_r'' = -\tfrac{1}{2}q\dot{\beta}''\cos^2\theta\, d\tau, \quad da_\theta'' = \tfrac{1}{2}q\dot{\beta}''\sin\theta\cos\theta\, d\tau,$$

so that all derivatives of $d\boldsymbol{a}''$ containing da_ϕ or D_ϕ vanish leaving

$$\nabla \times (d\boldsymbol{a}'') = \boldsymbol{k}_\phi \left\{ \frac{da_\theta''}{r} + \frac{1}{2r} q\dot{\beta}'' d\tau \frac{d\cos^2\theta}{d\theta} \right\}$$

or

$$d\boldsymbol{b}'' = -\tfrac{1}{2}\boldsymbol{k}_\phi \frac{q\dot{\beta}''d\tau}{r}\sin\theta\cos\theta. \tag{21}$$

Since when the system is unaccelerated

$$d\boldsymbol{b} = \boldsymbol{k}_\phi \frac{q d\tau}{r^2} \sin\theta$$

the effect of the introduction of $d\boldsymbol{b}''$ is like that of moving the element $d\tau$ in the direction opposite to that of $\boldsymbol{q}$ by an amount $\frac{1}{2}\dot{\boldsymbol{\beta}}'' r^2$ only inasmuch as this motion decreases θ by $\frac{1}{2}\dot{\beta}'' r \sin\theta$ thus changing $d\boldsymbol{b}$ by an amount

$$\boldsymbol{k}_\phi q d\tau \left\{ -\frac{\cos\theta}{r^2} \cdot \tfrac{1}{2}\dot{\beta}'' r \sin\theta \right\} = d\boldsymbol{b}''.$$

The complete effect of this imagined motion of $d\tau$ would include also a change of r and produce 3 times as large a change in the magnetic field. Thus the force on one element due to another accelerated in its own direction is affected by the acceleration of the system only inasmuch as the effective direction—not distance—from each one to the other has its changes due to acceleration delayed by the time r.

For the r component of $\dot{\beta}'''$ we have

$$\dot{\beta}''' \cos\varphi \sin\theta,$$

so that

$$da_\phi''' = 0, \quad da_r''' = -\tfrac{1}{2}\dot{\beta}''' q \cos\theta \sin\theta \cos\varphi \, d\tau,$$

$$da_\theta''' = +\tfrac{1}{2} q \dot{\beta}''' \sin^2\theta \cos\varphi \, d\tau.$$

Here φ derivatives do not vanish, and we have

$$d\boldsymbol{b}''' = \tfrac{1}{2}\boldsymbol{k}_r \frac{q\dot{\beta}''' d\tau}{r} \sin\theta \sin\varphi + \tfrac{1}{2}\boldsymbol{k}_\theta \frac{q\dot{\beta}''' d\tau}{r} \cos\theta \sin\varphi + \tfrac{1}{2}\dot{\boldsymbol{k}}_\phi \frac{q\dot{\beta}''' d\tau}{r} \cos^2\theta \cos\varphi \quad (22)$$

$$= \tfrac{1}{2} \cdot \frac{q\dot{\beta}''' d\tau}{r} \{\sin\varphi(\boldsymbol{k}_r \sin\theta + \boldsymbol{k}_\theta \cos\theta) + \cos\varphi(\boldsymbol{k}_\phi \cos^2\theta)\}.$$

Now let us compute the change in the vector $d\boldsymbol{b}$ produced by the changes of direction involved in a displacement of the element $d\tau$ in the direction opposite to $\dot{\beta}'''$, that is, of the observing point in the direction of $\dot{\beta}'''$, by an amount $\frac{1}{2}\dot{\beta}''' r^2$. These changes are of two sorts, first, those of $\sin\theta$, and second those of the direction $\boldsymbol{k}_\phi$.

For the first, we have an increase of θ by an amount $\frac{1}{2}\dot{\beta}''' r \cos\theta \cos\varphi$, which changes $d\boldsymbol{b}$ by exactly $d\boldsymbol{b}_\phi'''$.

For the second, we have a change of the direction of $\boldsymbol{k}_\phi$ by an angle $\frac{1}{2}\dot{\beta}''' r(\sin\varphi/\sin\theta)$, so that this vector is changed by the introduction of a component of this magnitude directed away from the axis of the coördinate system, so that its r component has this magnitude times $\sin\theta$,

and its θ component this magnitude times $\cos\theta$. Substituting these components for the $\boldsymbol{k}_\phi$ of the expression for $d\boldsymbol{b}$ we find increments in $d\boldsymbol{b}$ exactly equal to the r and θ components of $d\boldsymbol{b}'''$.

Thus these changes of $d\boldsymbol{b}$ due to the acceleration component perpendicular to q can also be identified with those produced by a delay by the time r in the changes of direction but not of distance from the point at which $d\boldsymbol{b}$ is evaluated to the element $d\tau$. The same statement therefore applies to all magnetic forces between any pair of volume elements.

Thus it appears that the magnetic forces on any element in such a system are always the same as if each other element was situated at the distance it actually is at but in the direction it would have if it had not been accelerated since the last radiation left it that has already arrived at the element in question. This is just the law postulated for the corresponding internal forces. Hence it appears that the part of the mass due to the magnetic field will not only have all dependence on direction of acceleration compensated by the internal forces, but will actually be balanced entirely by them. This completes the proof of the theorem and corollary stated above.[1]

This result is of especial interest as applied to the Parson magneton, since it signifies that it is not unreasonable to assume that the magnetic energy of the magneton can not be deduced in any way from its mass, but that the mass is due entirely to its electrostatic energy and the internal forces associated with it. As one may readily see, if the internal forces of either kind are assumed independent of the distance between the elements under all circumstances, the energy of each force, reckoned from zero distance is equal in absolute value to the mutual electrostatic or magnetic energy, but with a plus sign in the former case and a minus in the latter. Hence in this special case we may apply the conclusion so often drawn, though not proved, from relativity, that the mass of each part of the system is equal, in the units used here, to its energy.

[1] A curious paradox arises from the fact that in the case of motion of a magneton perpendicular to its axis the electromagnetic momentum (from equation 14) per unit length of the ring is twice as great in the part of the ring lying parallel to the line of motion as in the part perpendicular to it, and yet during acceleration the former part is subject to no retarding force from the magnetic field. At first sight this would lead one to suspect an error on one side or the other. But the proof of (14) is very direct, and the other assertion rests simply on the fact that the force exerted by any magnetic field on an element of current is perpendicular to the current.

For the solution of this problem it appears that the retarding force on the part of the ring accelerated perpendicular to itself is much greater for acceleration perpendicular to the axis than along it. While the two parts of the ring could exist independently so far as the laws of electrostatics are concerned, the fact that a steady current is impossible without a closed circuit makes it necessary to consider the whole ring in any complete analysis of the magnetic mass.

GYROSCOPIC EFFECTS IN THE MAGNETON.

The question of the existence or non-existence of gyroscopic effects when the magneton is rotated about an axis lying in its plane is given considerable importance by the experiments of Barnett[1] on the magnetization of a bar of iron by rotation, and of Einstein and de Haas[2] on the converse of this effect. Although both effects are very small and require extremely delicate measurements, these experiments are quantitative as well as qualitative and indicate that the gyroscopic effects of the magnetic particle are of the order of magnitude of those of an electron revolving in an orbit. As Parson[3] has pointed out there are serious difficulties in the w y of the assumption that magnetism is due to an orbital motion of a electron or ring of electrons of the classical type. For aside from the well-known difficulties connected with radiation, there is the fact that non-coplanar orbits in an atom are unstable, and yet absolutely necessary if most elements, especially light ones, are not to be strongly paramagnetic. Hence arises the necessity for an examination of the gyroscopic properties of the magneton.

Such a calculation is reported in a paper which I presented at the meeting of the American Physical Society in February, 1917, and of which an abstract will be found in the PHYSICAL REVIEW of 1917. The net result was that the gyroscopic properties of the magneton are exactly those of a classical electron in an orbit having the same magnetic moment, provided that the assumptions of the present paper are correct.

THE EXPLICIT FORMULA FOR THE MASS.

The radius of the cross section of the ring, or "small radius," R', as we shall call it, is extremely minute compared to the radius, R, of the ring itself. This simplifies greatly the calculation of the absolute value of the electrostatic energy, which would otherwise be quite difficult. One reason is that it makes the charge distribute itself practically uniformly around the cross section. Another is that the potential at any point on the center of the cross section will be practically the same as if all the charge were on the center line of the ring except for a length on each side of this point that is so short that it may be considered straight. While the truth of these assumptions is not self-evident, it may be proved in the calculation. For calculation, let the assumed straight piece have a length $2xR$, where x is a very small fraction and let $a = R'/R$. The line density of electricity on the ring is $e/2\pi R$. The potential at a point on the center line of the ring is

[1] Science, *30*, p. 413, 1909; PHYS. REV., *6*, 239–70, Oct., '15.
[2] Deutsch. Phys. Gesell., Verh., 17.8., pp. 152–70, Apr. 30, '15.
[3] L.c.

$$V = \frac{e}{2\pi R}\left[2\int_0^x \frac{Rd\theta}{\sqrt{R^2\theta^2 + R'^2}} + 2\int_x^\pi \frac{Rd\theta}{2R\sin\frac{\theta}{2}}\right], \tag{23}$$

where the first integral gives the effect of the straight cylinder (treated as having a surface charge) and the other gives that of the rest of the ring. The justification for our assumptions must be the cancellation of all x's from the result. By direct integration this becomes

$$V = \frac{e}{\pi R}\left[\log\frac{x + \sqrt{x^2 + a^2}}{a} + \log\frac{\tan\frac{\pi}{4}}{\tan\frac{x}{4}}\right]$$

$$= \frac{e}{\pi R}\log\left[8a^{-1}\frac{1 + \frac{1}{4}\frac{a^2}{x^2} + \cdots}{1 + \frac{1}{48}x^2 + \cdots}\right].$$

The x terms may now be dropped with no appreciable error, since the fraction a is so inconceivably small that a/x and x can both be negligible at the same time. This leaves

$$V = \frac{e}{\pi R}\log\frac{8R}{R'}.$$

Since the potential is constant within and on the surface of the ring, the electrostatic energy is

$$\tfrac{1}{2}eV = \frac{e^2}{2\pi R}\log\frac{8R}{R'}. \tag{24}$$

(It may be worth noticing that the magnetic energy may be found by Rayleigh's self-inductance formula[1] as

$$\frac{e^2}{2\pi R}\frac{v^2}{c^2}\log\frac{8R}{R'}$$

or v^2/c^2 times the electrostatic energy, as we should expect.)

Thus the mass of the magneton in ordinary units is

$$m = \frac{eV}{c^2} = \frac{e^2}{\pi c^2 R}\log\frac{8R}{R'}. \tag{25}$$

For the small radius R', this gives the formula

$$R' = 8Re^{-\frac{\pi Rmc^2}{e^2}}. \tag{26}$$

Since $e^2/mc^2 = 2.82 \times 10^{-13}$ cm., this makes R' almost incredibly small if R is as large as the value 1.5×10^{-9} cm., assigned by Parson. This,

[1] Roy. Soc. Proc., 32, p. 104, 1881; *A* 86, p. 562, 1912.

however, is very uncertain, so no definite value of R' can be assigned at this time.

SUMMARY.

1. It is shown that the relativity principle requires certain assumptions about the internal forces in the magneton, or any other non-spherical system, producing a sort of mass which, added to the electromagnetic mass, will make the total mass constant.

2. Making these assumptions in the most plausible way, the mass of the magneton is $2/c^2$ times its electrostatic energy, with no reference to its magnetic properties.

3. This result leads to a formula for the mass from which the small radius could be determined if the large radius were accurately known.

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REMARKS ON A PAPER BY MR. E. R. STOEKLE ENTITLED "THERMIONIC CURRENTS FROM MOLYBDENUM."

BY O. W. RICHARDSON.

IN this very interesting paper published in the PHYSICAL REVIEW for November, 1916, Mr. Stoekle refers to some experiments I made with tungsten filaments, an account of which was published in the Philosophical Magazine (Vol. 26, p. 345, 1913). After pointing out that I made certain experiments which showed that the electron emission could not originate from any process involving the consumption of the hot filament, he says: "His conclusions, however, as to the absence of any appreciable change in the electron emission even upon allowing 'considerable changes in the amount and nature of the gases present in the tube' are not in accord with the observations of I. Langmuir upon the thermionic emission from tungsten nor with the results of the experiments upon molybdenum to be set forth in this paper."

This statement involves a misunderstanding of the language of my paper. None of my conclusions, so far as I am aware, contravene in any way the observations either of Dr. Langmuir or of Mr. Stoekle.

There are two passages in my paper which appear to be particularly relevant to the point at issue. On p. 347 I point out that in a vacuum in which the pressure, apart from mercury vapor, was of the order 10^{-7} mm. the electron current had the same value whether mercury vapor at a low pressure was present or not, and that a difference of 0.4 per cent. in the current, due to this cause, would have been detected. After showing that the emission is not to be attributed to any of the following alternative causes, viz.: (1) evolution of gas by the filaments, (2) chemical action or some other cause depending on impacts between the gas molecules and the filaments, and (3) some process involving consumption of the tungsten, I say on p. 350:

"(4) The only remaining process of a similar nature to those already considered which has not been discussed is the bare possibility that the emission is due to the interaction of the tungsten with some unknown condensable vapor which does not affect the McLeod gauge. This possibility is cut out by the fact that the thermionic emission is not affected when the liquid air and charcoal is cut off and the vapors allowed

to accumulate in the tube, and by the fact that very considerable changes in the amount and nature of the gases present (as by the admission of mercury vapor) have no effect on the emission."

In interpreting this passage it is to be remembered that in writing it I was arguing against a supposititious theory in which I did not believe and for the correctness of which there was no evidence, and that, apart from the allusion to mercury vapor, the vapors and gases referred to are purely hypothetical and imaginary. So far as the effect of mercury vapor on the emission from tungsten is concerned I would direct attention to the following quotation from a recent paper by Dr. Langmuir[1]: "Hydrogen, mercury vapor and the inert gases, when pure, have no effect on the electron emission."

[1] Trans. Amer. Electrochem. Soc., p. 352 (April, 1916).

BINAURAL BEATS.

BY G. W. STEWART.

THE first recorded experiments with binaural beats seem to have been made by Dove[1] in 1839. He found that beats were obtained with two forks in unison by separating the forks with one near each ear as well as by presenting both of them to one ear. Obviously these and indeed all binaural experiments have a considerable interest to the physicist, the physiologist, the anatomist, and the psychologist. This breadth of interest has resulted in a scattering of the literature and in serious omissions of recognition of earlier work. Paul Rostosky[2] has prepared an apparently thorough review of the experiments concerning the results obtained by and conditioned upon bilateral excitation of the organs of hearing. The review includes all results obtained up to and including the year 1896. The excellence of Rostosky's résumé enables the writer to abbreviate greatly references to the work therein described. The policy has been adopted in this paper of accepting results that are amply confirmed by relatively recent workers, without attempting to explain the earlier disagreements which were probably due to individual differences and to the limitations imposed by the particular forms of apparatus.

The chief phenomena observed when two tones differing in frequency are led one to each ear are as follows:[3]

(*a*) Beats occur whether the tones are presented to one ear or one to each ear. This was discovered by Dove,[4] observed subsequently by various experimenters and discovered independently by S. P. Thompson.[5] This phenomenon is observed by all, even by the partially deaf.

(*b*) With bilateral presentation of the tones, the minimum intensity observed in each beat does not (even with carefully adjusted equal intensities of the two sources) approximate zero. This is mentioned by S. P. Thompson[6] but doubtless was observed very much earlier by others.

[1] H. W. Dove, Repertorium d. Physik, Bd. 3, p. 404, 1839.

[2] Paul Rostosky, Beiträge zur Psychologie und Philosophie, I, 1905, pp. 173–273.

[3] Unless otherwise stated, the difference of frequencies is assumed to be small.

[4] *Loc. cit.*

[5] S. P. Thompson, Phil. Mag., 5, IV., 1877, p. 274.

[6] *Loc. cit.*

(*c*) When the tones at the ears are alike in intensity and phase, there is a localization in the median plane, the position in this plane varying with the observer and presumably with the apparatus arrangement. Purkyně[1] was the first to record this localization, which has been shown through numerous experiments of other observers, to be a usual accompaniment of binaural perception.

(*d*) With opposition in phase and equality in intensity, the sound is localized according to S. P. Thompson[2] at the back of the head. To several observers, with apparatus of a form different than that of Thompson's the localization is in the median plane. Doubtless the position in the median plane is influenced by the method of observation.

(*e*) The sound is localized on the side of the fork leading in phase when the phase difference is approximately 90°. Thus with phase differences the sound wanders from the median plane at 0° to one side at 90°, to the median plane at 180°, to the other side at 270° and again to the median plane at 360°. Perhaps S. P. Thompson[3] first observed the wandering from ear to ear but the periodic displacement of the localization was first most clearly described by Paul Rostosky.[4] Later workers, who were not aware of Rostosky's results, observed this lateral displacement. Lord Rayleigh[5] experimenting with forks, recognized the lateral effect and its relation to phase difference, and later[6] confirmed these observations with experiments conducted by the use of telephones. L. T. More and H. S. Fry[7] made preliminary experiments which were followed by those of L. T. More.[8] His results showed unmistakably the lateral displacement. More used an apparatus similar to that earlier adopted by C. S. Myers and H. A. Wilson.[9] A single source, branched pipes to the ears and an arrangement whereby the lengths of the pipes could be altered in opposite senses, were used. The experiments of Myers and Wilson also gave full confirmation of the lateral effects. Bowlker's[10] experiments do not seem to furnish reliable evidence. All the workers just mentioned were not aware of the splendid experiments of Rostosky published in 1902.[11]

(*f*) There is a limit to the frequency of the tone with which the

[1] Purkyně, Prager Vierteljahrsschrift, Bd. 67, p. 91, 1860.
[2] S. P. Thompson, Phil. Mag., 5, IV., 1877, p. 274, and Phil. Mag., 5, VI., 1878, p. 383.
[3] S. P. Thompson, Phil. Mag., 5, XII., 1881, p. 351.
[4] P. Rostosky, Philosophische Studien, 19, 1902, p. 557.
[5] Lord Rayleigh, Phil. Mag., XIII., 1907, p. 214.
[6] Lord Rayleigh, Phil. Mag., XIII., 1907, pp. 316–319.
[7] L. T. More and H. S. Fry, Phil. Mag., XIII., 1907, p. 452.
[8] L. T. More, Phil. Mag., XVIII., 1909, p. 308.
[9] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., LXXX., 1908, p. 260.
[10] T. J. Bowlker, Phil. Mag., (6) XV., 1908, p. 318.
[11] *Loc. cit.*

effect just described (*e*) can be obtained. L. T. More[1] states that a fork of 512 d.v. is near to his limit of accuracy in the judgment by phase differences, that with 1,024 d.v. his judgment became untrustworthy and that with 3,000 d.v. he had no sense whatever of direction. Lord Rayleigh[2] gives the limiting frequency of his right and left sensations as 768 d.v. C. S. Myers and H. A. Wilson[3] state in general terms that with very high frequencies the lateral effects cannot be obtained.

(*g*) In addition to the maximum at 0° phase difference, there are also two additional maxima occurring one before and the other after opposition in phase, but distinctly perceptible only when the beats are less frequent than one in two to five seconds. These additional maxima were first studied by Paul Rostosky[4] in 1902, but in the succeeding literature no reference is made to his important contribution. Thus the subsequent discovery of these secondary maxima by the writer was an independent one. The results of the investigation of the latter are presented in this report.

(*h*) There are several other phenomena of importance, such as the effects obtained by placing forks on the skull, by using forks of large frequency differences, the result of practice of one ear upon the acuteness of the other, and the binaural audibility of two tones each one of which alone may be subliminal. For the present, we will be contented merely with this mention of them.

APPARATUS.

Mirrors were attached on the prongs of tuning forks so that the plane of each mirror was perpendicular to the vibration of its prong. The forks were then mounted so that their vibrations were in perpendicular planes. The light from an arc lamp, by the use of lenses and an aperture, was reflected from two of the mirrors, one on each fork, and formed a small spot of light or an image of the aperture, upon a screen. With vibration of the forks, Lissajou's figures were traced upon the screen, and by these the phase relation of the forks could be ascertained. Glass tubes were mounted, one close to a prong of each fork and were connected by means of two pieces of rubber tubing each a meter in length, to ear tubes such as are used with stethoscopes. In certain experiments the manner of introducing the sound to the ear was changed, but of this mention will later be made. The forks were operated electrically by using a short helical spring contact, 0.3 cm. in radius, of platinum

[1] *Loc. cit.*

[2] Lord Rayleigh, Phil. Mag., XIII., 1907, p. 214.

[3] *Loc. cit.*

[4] *Loc. cit.*

wire, No. 29 B. & S. gage. The contacts were very quiet and gave a remarkable constancy of amplitude. Indeed the writer has observed the amplitude of a fork so actuated to remain constant, within one per cent., for an hour. No difficulty was experienced because of the presence of both forks in the same room. After many months' work with the apparatus, experimenting upon numerous individuals and confirming completely the experiments of Rostosky and others wherein even greater precautions were taken, the writer can confidently state that the results are in no wise dependent upon any "stray" sounds. Indeed, all the important results were confirmed by leading the tubes into an adjacent room where the forks were not at all audible to the unstopped ears. In practically all the experiments in the same room with the forks, the latter were scarcely, if at all, audible save when the tubes were placed in the ears.

THE RESULTS.

The following results were obtained with the assistance of Mr. F. C. Bruene.

1. The beats with the tones of two forks, one applied to each ear, were heard distinctly. Indeed, of twenty-three inexperienced observers listening for three periods of five minutes each, seventeen noticed the beats distinctly enough to report them. All experienced observers can hear them and record them with very satisfactory accuracy.

2. There existed a distinct wandering of the localization; in front at 0° difference of phase, on the side of the fork with the higher frequency leading in phase from 0° to 180°, and on the side of the slower fork with the higher frequency leading in phase from 180° to 360°, the changes of position being continuous, from front to rear (or within the head) and from rear to front. This wandering when first recognized might be described as occurring in a circular path. A more accurate description occurs in what follows.

3. This rotation of the localization just described was quickly and distinctly observed by some hearers; on the other hand all observers did not recognize it, even with an hour or more of practice. Eleven of the twenty-three uninstructed and inexperienced observers reported their observation of this "rotation" after fifteen minutes' trial (three trials of five minutes each). Six of these eleven noticed the effect within the first five minutes, and three in the second.

4. All of the eleven just mentioned, with the possible exception of one or two, noticed that the localization was much more distinct in the half cycle in which the lead of the faster fork varies from $-90°$ to $+90°$. With experienced observers, the effect is very noticeable.

5. Extended experiments with four experienced observers showed that the most accurate judgment as to phase difference, made by the localization, occurred when that difference is 0°.

6. With the experienced observers the change in the localization at 0° phase difference by the production of a very marked inequality in intensities, is slight. The rotation of the localization was modified by a slight circular shift in the direction of the side of the greater excitation. The effect upon the localization at the sides became very marked, that on the side of the greater intensity becoming more prominent and that on the other almost vanishing.

7. With observers of experience, but who had not yet recognized the secondary maxima,[1] the maxima intensity seemed to occur at 0° phase difference when the beat-period was less than perhaps two seconds. With a greater and gradually increasing period there arose an uncertainty until with a period of perhaps five seconds, the judgment of maximum intensity shifted to a phase difference noticeably less than 180°, or approximately to the first occurring secondary maximum.

8. The rotation of localization with experienced observers occurred as follows: At 0° phase difference the sound was localized as in front and distant, *i. e.*, external to the head. The localization then described in a horizontal plane a path which appeared to be somewhat circular, but in which the apparent distance contracted until the path entered the ear leading in phase. Then at about 180° phase difference the localization passed quickly through the head, into the other ear and from thence around to the front along a path symmetrical to the one just described.

9. When an experienced observer gave his attention to apparent intensity rather than to localization, there appeared three fairly distinct maxima. One occurred at 0° difference of phase, one at $180° - \delta$ and one at $180° + \delta$, δ being less than 45°. The earlier of the additional maxima at $180° - \delta$ phase difference, coincided with the localization in the ear near the higher frequency, and the later one with the localization in the other ear. With a 128 d.v. fork the secondary maxima seemed clearly discernible only if the beat-period exceeded a value of approximately two seconds. The value given is only approximate as the variation in the perception of the secondary maxima is continuous with the changing of the beat-frequency. The significant fact is that the 0° maximum was always present, whereas the secondary ones required a beat-period exceeding at least a second in order to become clearly evident.

10. The localization in the ears as described seemed to depend upon

[1] See (*g*) and (9).

the perception of the secondary maxima. If they were absent, as at high beat-frequency, the localization seemed confined to the region near the front. In other words the localization in approximately one half the cycle seemed to be due to the secondary maxima or the causes producing them.

Discussion of Results.

All of the results are not new. (1) was first discovered by Dove as already stated. (2) was in part reported by S. P. Thompson,[1] Lord Rayleigh,[2] C. S. Myers and H. A. Wilson,[3] and L. T. More.[4] Paul Rostosky was the first to record (2) and (8) and (9) completely. As previously stated in this article, Rostosky's work was unknown to Lord Rayleigh, C. S. Myers and H. A. Wilson, and L. T. More. Thus it chanced that the writer's discovery of the secondary maxima and the rotary character of the localization was an independent one.

None of the phenomena described owed its origin to sounds or noises other than the tones of the forks. Neither did any sound enter the left ear by aerial conduction from the tube entering the right ear. The former statement was tested by conducting the tubes to an adjoining room and by the many alterations in apparatus necessitated by the changing of the forks in experiments described in a succeeding article. The latter statement was verified by the use of tones of very faint audibility and by changing the character of the introduction of sound to the ear (*i. e.*, by the use of binaurals that are not inserted in the ears), without changing the phenomena observed in either case. Reference to Rostosky's experiments will show that he took much more elaborate precautions. Yet our results are in agreement with his at every point. The evidence thus favors the judgment that our precautions were sufficient.

There is an objection to the use of binaurals inserted in the ears, for by partially closing the external meatus they emphasize sounds that are conveyed from the skull to the volume of enclosed air. From the physical point of view, this partial closing of the external meatus would not be likely to change the characteristics of the phenomena and experiment shows this to be the case. Indeed, Rostosky used binaurals that were not inserted into but merely presented in front of the opening of the ear. The writer was also able to detect all the phenomena without the use of inserted binaurals. But there is an additional objection to the inserted binaurals, for there is the chance of conduction from the terminal of the binaural to skull. Experiments in stopping the openings

[1] *Loc. cit.*
[2] *Loc. cit.*
[3] *Loc. cit.*
[4] *Loc. cit.*

of the binaurals showed that there remained only an exceedingly faint trace of sound which could be fully explained by the failure of the stoppage. The conclusion of the writer is that the results with inserted and with non-inserted binaurals are not qualitatively different.

The results given point definitely to distinct differences between the causes of the localization in front and in the rear, or in the ears. The two halves of the cycle differ in distinctness of localization, in independence of beat frequency, in effects produced by inequalities of intensities, and in the apparent distance from the head, the localization in one half being external and in the other half internal. These differences must be accounted for in any adequate theory.

The fact that some observers by the single source method of C. S. Myers and H. A. Wilson[1] do not obtain such a clearly defined "rotation" but rather a wandering back and forth from the median plane to the sides, is apparently due to the difference in the method of observing. In our experiments, with the continually changing phase difference, the rotation is observed in every case where localization is obtained.

This article gives the chief phenomena characteristic of binaural beats. In succeeding articles will be presented a study of the secondary maxima and a physical theory of all the phenomena involved. It will appear that the experiments do furnish excellent evidence as to the perception of phase by the ear and as to a two-fold nature of the hearing organ.

I wish to acknowledge my indebtedness to Mr. F. C. Bruene for his assistance in the experimental work.

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[1] *Loc. cit.*

THE SECONDARY INTENSITY MAXIMA IN BINAURAL BEATS.

BY G. W. STEWART.

THIS paper is a continuation of a consideration of binaural beats;[1] and contains the results of a study of one of the phenomena involved, namely, the secondary intensity maxima. The physics of hearing has been and is an important study, not because the discovery of new principles in physics are to be expected, but because the progress in the study of audition in the related sciences of psychology, anatomy, and physiology, is dependent upon a clear understanding of the physical aspects involved.

The reader should recall the fact[2] that when two tones of almost the same pitch are presented one to each ear, there appears an intensity maximum with 0° difference in phase, and secondary maxima at phase differences of $180° - \delta$ and $180° + \delta$. The secondary maxima are perceived clearly only when the beat-period exceeds one or two seconds. They seem to have an origin somewhat different than that of the zero-phase maximum, and therefore a study of them is important.

The results herein presented were obtained with the apparatus already described.[3] There were two observers, one to operate the stop-watch at a chosen signal from the other who listened for the appearance of the secondary maxima. The method was to measure the time interval between the two secondary maxima, beginning with the first secondary maximum following zero-phase difference. Thus the interval included opposition in phase, and measured the time for a phase difference change of 2δ.

Results.—The secondary maxima will be considered as occurring at $180° - \delta$ and $180° + \delta$ phase differences, for to an observer watching the Lissajou's figures, it is evident that the phase difference of 180° occurs midway between the secondary maxima. The question arises, does δ change with the frequency of beats? The answer can be found in the following Table I.

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 502–508.

[2] See *loc. cit.*

[3] *Loc. cit.*

TABLE I.

Forks of 128 d. v.

Date.	Period of Beats.	2δ in Sec.	2δ/2π.	2δ/2π.
Aug. 3	27.8		.20	
" 3	12.3	2.60(7)		.21
" 3	12.2	3.50(7)		.29
" 3	13.1	2.75(8)		.21
" 10	35.7	7.9 (4)	.22	
" 10	35.5	7.2 (6)	.20	
" 10	36.0	10.3 (9)	.29	
" 10	13.7	2.95(12)		.22
" 10	14.0	3.6 (11)		.26
" 11	35.2	7.9 (4)	.22	
" 11	13.3	2.68(10)		.20
" 11	13.5	3.1 (6)		.23
Mean.....	.....	.....	.23	.23

The number in parenthesis in the third column indicates the number of actual measurements, the average of which is expressed in the number just preceding. This table shows that with the beat-period increased by the factor three, no definite change takes place in δ. Similar results

TABLE II.

Forks of 42, 63, and 128 d. v.

Fork of 42 d.v.			Fork of 128 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
Aug. 11	20.4	.19 (10)	June 27	23.2	.25 (5)
" 11	20.2	.20 (8)	" 27	10.1	.29 (3)
Aug. 12	22.6	.18 (10)	June 29	23.9	.24 (3)
" 12	23.7	.16 (8)	" 29	31.0	.22 (3)
" 12	22.6	.18 (9)	" 29	16.0	.27 (4)
" 12	24.5	.15 (11)	July 2	35.8	.22 (4)
" 12	21.6	.17 (13)	" 3	27.8	.20
" 12	21.5	.18 (10)	" 3	12.3	.21 (7)
Mean.....	.....	.17	" 3	12.2	.29 (7)
Fork of 63 d.v.			" 3	13.1	21 (8)
			" 10	36.0	.29 (9)
Aug. 3	10.1	.20 (11)	" 10	35.5	.20 (6)
"	10.7	.20 (13)	" 10	35.7	.22 (9)
"	10.8	.18 (11)	" 10	13.7	.22 (12)
Aug. 5	10.2	.18 (4)	" 10	13.7	.25 (10)
"	19.5	.16 (6)	" 10	14.0	.26 (11)
"	19.0	.14 (4)	" 10	13.6	.20 (7)
Weighted mean.....		.18	" 11	13.3	.20 (10)
			" 11	13.5	.23 (9)
			" 11	35.2	.22 (4)
			Mean	.....	.23

were obtained with forks of different frequencies, and can be inspected in succeeding tables where they are presented for another purpose. The data show the inaccuracy of the measurements of 2δ. A maximum would naturally be difficult to locate, for the hearing sense cannot detect small changes in intensity. But, as the succeeding data will show, this is not the only cause for inaccuracy. The constancy of δ with the frequency of beats is an important consideration in the theory of the secondary intensity maxima.

In what manner does δ vary with the frequency of the forks? Pairs of forks having approximately 42, 63, 128, 256, 384 and 469 d.v. were used in the experiments. The results are shown in Tables II. and III.

TABLE III.

Forks of 256, 361 and 469 d. v.

Fork of 256 d.v.			Fork of 361 d.v.		
Date.	Beat Period.	δ/π	Date.	Beat Period.	δ/π
July 13	13.2	.26 (9)	June 26	15.9	.28 (9)
" 13	12.2	.29 (9)	" 26	8.2	.29 (10)
" 14	43.8	.26 (4)	July 20	17.6	.28 (6)
" 14	27.4	.27 (8)	" 20	22.3	.26 (7)
" 14	23.4	.21 (8)	" 20	17.1	.25 (8)
" 14	12.0	.28 (11)	" 27	10.7	.27 (9)
" 14	10.5	.26 (9)	" 27	10.2	.24 (15)
" 14	9.25	.27 (9)	" 27	11.7	.30 (5)
Mean.....	.....	.26	" 27	9.1	.28 (9)
			" 27	12.5	.22 (7)
Fork of 469 d.v.			Aug. 1	12.7	.30 (6)
Aug. 5	4.2	.32 (1)	Mean.....	.....	.27
"	4.0	.37 (2)			
"	6.6	.27 (4)			
"	5.6	.35 (2)			
"	5.0	.38 (2)			
"	5.0	.26 (3)			
"	7.0	.27 (3)			
Mean.....	.....	.32			

Here in the third and sixth columns are given the values of δ as fractions of π. The number in parenthesis following each value gives the number of measurements of which that value is the average. The data indicate conclusively the inaccuracy of the measurement. They show that with the same forks and the same beat-frequency, but with observations made on different days, there is a considerable variation. It was found that observations made at different times of day with the observer in different physical states gave different results. Consequently, the observations

on each day were scattered as widely as possible. All of the observations recorded were made by the writer, assisted by Dr. Harold Stiles of the Iowa State College.

Fig. 1 is presented in order to show the character of the variation of δ with the frequency of the forks. While the results are not accurate, the fact that the relation is not linear is indubitable. δ increases less rapidly than in proportion to the frequency.

The reader should be reminded that in all of the observations herein reported the two tones were presented by means of stethoscope-binaurals which were inserted in the external meatus of the ear. Objections to this method are stated and answered in the previous article.[1] It is the opinion of the writer that while the quantitative results here presented might be different with open binaurals, yet the conclusions to be drawn from our experiments are not thereby affected.

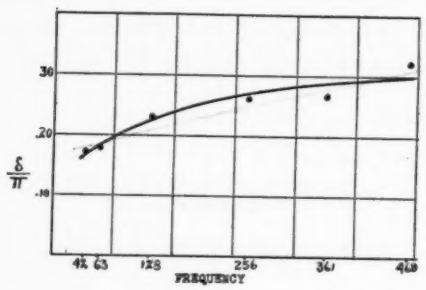

Fig. 1.

Conclusion.—A moment's consideration of the presence of the secondary maxima leads one to assume that the effect is due to conduction from one ear to the other through the head. The first result, viz., the fact that with a given fork frequency δ is independent of the beat-frequency, is distinctly in harmony with this assumption. For, suppose an indefinite number of conduction paths in the skull from the aërial vibration on one side to the ear of the other side. The resultant of these conducted waves will differ in phase from the original aërial vibration by an amount which depends upon the conducting paths and the frequency of the vibration. For any given fork frequency this introduction of phase is therefore constant. (It will be shown in the theory to be published in this journal that this phase is, in fact, δ.) Hence δ should be independent of beat-frqeuency. Thus the expectation that the explanation of the secondary maxima is to be found in physical theory receives encouragement.

The second result, which shows a nonlinear variation of δ with fork-frequency, might seem in disagreement with physical theory, for with any path between the ears, the retardation in phase by propagation thereon would be proportional to the frequency. This is true for a single path; it is true for a duplex path, but, as can be readily shown, only in

[1] G. W. Stewart, *loc. cit.*

case the amplitudes in these paths are equal. In as much as the amplitudes in all the conducting paths could not be equal, these paths cannot be equivalent to a single path of constant length. Hence we see that the two results herein described are not inconsistent with a theory which is based upon skull conduction from one side to the opposite organ of hearing.

I wish to acknowledge my indebtedness to Dr. Harold Stiles for his assistance in the observational work.

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THE THEORY OF BINAURAL BEATS.

BY G. W. STEWART.

AS described in preceding articles on binaural beats,[1] there are several phenomena that invite an explanation in terms of physics. The three outstanding aspects are (1) the existence of the "binaural" beats, herein referred to as "primary" beats which correspond to these heard uniaurally with two forks almost in unison, (2) the distinct perception of secondary intensity-maxima when the binaural beat-period is greater than two to five seconds, and (3) the wandering localization of the sound during each beat. We will consider these three separately and propose a theory which seems to promise a satisfactory explanation.

PRIMARY BEATS.

Dove[2] in 1839 discovered that when two forks, almost in unison, were presented first to one ear and then, by carrying one about the head, one to each ear, the beats perceived in the first position disappeared and subsequently reappeared as the moving fork approached its final position. I have chosen to call these binaural beats the "primary" beats, for they are always present with observers who are not seriously deaf in one ear. This is not true of the "secondary" beats which occur with clearness only if the primary beat-period exceeds two to five seconds. Separate treatment of the primary beats is justifiable, and in order to make the theory conform strictly to the fact, let us suppose we are dealing with a primary beat-period of approximately one second, or a period where the primary beats alone appear. As already described,[3] we have then a maximum with 0° and a minimum with 180° phase differences.

Let us assume that the forks F_1 and F_2 are presented one to each ear, that their amplitudes are equal, that F_1 has the greater frequency, and that there are conducting paths from each fork to the farther ear.

Let ϵ_1 be the retardation in phase in transmission from each fork to the nearer ear. (By the "ear" is meant the physical instrument producing the sensation.)

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, p. 502 and p. 509.

[2] H. W. Dove, Repertorium d. Physik, Bd. 3, p. 404, 1839.

[3] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, p. 502.

Let ϵ_2 be the phase retardation in transmission to the farther ear.

Let θ be the phase of the slower fork. $\theta = 2\pi ft$, where f is the frequency.

Let $\theta + \epsilon$ be the phase of the faster fork.

Let α be the amplitude at the ear produced by the nearer fork.

Let β be the amplitude at the ear produced by the farther fork.

The the resulting displacement at the ear nearer to the faster fork, at the instant represented by the value θ, is $\alpha \cos(\theta+\epsilon-\epsilon_1)+\beta \cos(\theta-\epsilon_2)$. At the farther ear the displacement is $\alpha \cos(\theta-\epsilon_1)+\beta \cos(\theta+\epsilon-\epsilon_2)$. By expanding we find the former displacement to be $\sin\theta[-\alpha \sin(\epsilon-\epsilon_1)+\beta \sin\epsilon_2]+\cos\theta\,[\alpha \cos(\epsilon-\epsilon_1)+\beta \cos\epsilon_2]$ and the maximum displacement, or amplitude, to be

$$\sqrt{[-\alpha \sin(\epsilon-\epsilon_1)+\beta \sin\epsilon_2]^2+[\alpha \cos(\epsilon-\epsilon_1)+\beta \cos\epsilon_2]^2}.$$

The energy of the vibration is proportional to the square of the amplitude. Hence the energy at the ear nearer to the faster fork is,

$$E_1 = \alpha^2+\beta^2+2\alpha\beta \cos(\epsilon-\epsilon_1+\epsilon_2).$$

E_2, or the energy at the ear farther from the faster fork, can be written down by interchanging β and α and ϵ_2 and ϵ_1, or,

$$E_2 = \alpha^2+\beta^2+2\alpha\beta \cos(\epsilon-\epsilon_2+\epsilon_1).$$

Let $\epsilon_2-\epsilon_1$ be represented by λ.

Then

$$E_1 = \alpha^2+\beta^2+2\alpha\beta \cos(\epsilon+\lambda) \quad (1)$$

$$E_2 = \alpha^2+\beta^2+2\alpha\beta \cos(\epsilon-\lambda) \quad (2)$$

If the apparent intensity is determined by the sum of E_1 and E_2, as we will assume, then,

$$E_1+E_2 = 2(\alpha^2+\beta^2)+4\alpha\beta \cos\epsilon \cos\lambda \quad (3)$$

and the apparent intensity will be a maximum at $\epsilon = 0°$ and a minimum at $\epsilon = 180°$. Moreover, since in the very nature of the case β is less than α, the minimum can never be a silence. Thus the theory coincides qualitatively with experiment. Quantitative comparison seems impossible.

Discussion.—It will be observed that the above theory does not designate the points of interference in each organ of hearing, but that it does stipulate the occurrence of the interference in a wholly physical instrument. In other words, peripheral interference is accepted as the cause of the primary maxima, or the "beats." To this extent the theory is not new, for Dove,[1] the discoverer of the primary maxima,

[1] *Loc. cit.*

recognized peripheral interference as a possible explanation. This suggestion has often been repeated. Paul Rostosky,[1] in his theory of the localization and the secondary maxima accepts this non-synchronous variation of the resulting intensity in each ear. But neither in his excellent review[2] nor in his own contribution just mentioned does Rostosky give a detailed theory explaining the primary maxima and minima.

There may be an objection to the summation of intensities herein employed, but in justification the writer submits four considerations. First, we have the fact that two like tones, each alone faint enough to be just at the limit of hearing, become distinctly audible when simultaneously each is presented to an ear.[3] Second, summation seems reasonable when one considers the phenomena of the strengthening of intensity in the numerous binaural experiments recorded in Rostosky's review, just mentioned. Third, the experiments of the writer[4] upon the accuracy of localization of various frequencies, the head not remaining stationary, gives strong evidence in favor of the intensity-sum as an important factor. In these experiments the frequencies most accurately localized are those in which the theoretical variations of the intensity-sum are the greatest. Finally, the perfect blending of the two tones which is actually observed points distinctly to the summation. According to the theory just proposed, the effect is independent of beat-frequency, that is, the primary maxima and minima ought always to appear, which is in accord with fact. The above simple theory would seem to explain the primary maxima and minima satisfactorily. But the phenomena is much more complex, and these additional complexities will be discussed in a following section.

Peterson[5] discusses the "nature and probable origin" of primary beats and concludes that they are cortical rather than peripheral in origin. His reasons are based upon the evidence of Cross & Goodwin[6] to the effect that difference tones cannot be obtained binaurally, the evidence, from experiments on two people each deaf in one ear, that bone conduction must be very small, and upon certain experiments in the counting of the primary beats. The evidence offered by Peterson and the argument in favor of his conclusion are worthy of careful consideration. But in as much as he does not set up any physical theory, a discussion of his paper is probably not appropriate to this article. But the writer should

[1] P. Rostosky, Philosophie Studien, 1902, 19, p. 557.
[2] P. Rostosky, Beiträge Zur Psychologie and Philosophie, I., 1905, pp. 173–273.
[3] Tarchanow, St. Peterburger Med. Wochensclos., 1878, No. 43, (n.R.).
[4] G. W. Stewart, PHYS. REV., N.S., Vol. 2, No. 1, July, 1913.
[5] Joseph Peterson, Psychological Review, Vol. XXIII, No. 5, Sept., 1916.
[6] C. R. Cross & H. M. Goodwin, Proc. of Academy of Arts & Science, 1891, 27, 1.

call attention to the fact that the theory of this paper not only assumes a peripheral interference which Peterson denies, but in addition a summation of intensities or a blending which obviously cannot be peripheral.

The Secondary Maxima.

The only records of the appearance of the secondary maxima are those of Paul Rostosky[1] and the writer.[2] Rostosky seeks to explain these maxima which appear first in one ear and then in the other, by a consideration of the localization. He concludes that the ratio of the two intensities, represented by equations (1) and (2), determines the degree of the right or left localizations. If one assumes α and β to be of the same order, say 1 to $\sqrt{2}$ or 1 to 2 (the values used by Rostosky), and then plots the ratio of E_2, (2), to E_1, (1), he will obtain a curve similar to Fig. 1. The curve is interpreted by Rostosky as the degree of right localization, the lower half of the curve, inverted, being the degree of left localization. Thus the maximum right localization appears distinctly or extremely right and gives the impression of a maximum intensity. Hence the two secondary maxima occurring in the neighborhood of a difference of phase of 180° are explained qualitatively; indeed, there also seem to be no objections thereto on the basis of quantitative considerations. Unfortunately, however, Rostosky makes a fatal error in his theory. His equations similar to (1) and (2) are[3] as follows:

$$J_l = a^2 + a_1^2 + 2aa_1 \cos (x + d/2)$$
$$J_r = a^2 + a_1^2 + 2aa_1 \cos (x - d/2).$$

Where the subscripts l and r refer to the left and right sides respectively, and a, a_1, x and $d/2$ correspond to our α, β, ϵ and λ, respectively. d is a positive quantity, hence J_l reaches a maximum, with changing phase x before J_r, the former occurring at $x = -d/2$ and the latter at $x = d/2$. The faster fork is thus on the left. Moreover, between $x = 0°$ and $x = 180°$, J_l is less than J_r. Experimentally, the secondary maximum appears first (after $x = 0°$) on the side of the faster fork, and hence on the side of the less theoretical intensity. If the maximum in Rostosky's curve, Fig. 1, represents localization, then (accepting his equations just given) it must represent "left" localization. But this curve is the ratio J_r/J_l and therefore must be interpreted as "right" localization. Hence we find that experiment and his theory do not agree. His error is due to a confusion of sides; in fact, one is disposed to guess (incorrectly) that the side of greater intensity (when $0° < x < 180°$) is that of the faster fork.

[1] Paul Rostosky, Philosophie Studien, 1902, 19, p. 557.

[2] G. W. Stewart, Phys. Rev., 2d Series, Vol. IX., No. 6, 1917, 509–513.

[3] See *loc. cit.*, p. 580.

The writer has considered whether or not an additional assumption of a change in phase during the conduction across the head would lead to a revision of Rostosky's theory that would be in harmony with observation, and he finds a revision upon such an assumption impossible. We are thus led to a complete rejection of the theory of Rostosky.

The writer's theory of the secondary maxima is based upon the outstanding importance of the following experimental facts which have been described in the two preceding articles.[1]

1. The secondary maxima do not occur unless the period of the beats is greater than, say, two to five seconds, whereas the presence of the primary maxima is independent of the beat-period.

2. At the secondary maxima the sound seems localized in the ears (first in the one nearer the faster fork), and between the maxima within the head. With the primary maxima the localization is distinctly external to the head.

3. The sound seems to be much rougher at the secondary than at the primary maxima.

4. The movement of the localization is much more rapid between the two secondary maxima than elsewhere in the rotation.

5. If the secondary maxima appear at phase differences of $180° - \delta$ and $180° + \delta$ then, for a given fork pitch, δ is independent of the beat-frequency, but δ does vary with the frequency of the forks used, and this variation is not a linear one.

6. Unequal exciting intensities at the ears have a tendency to eliminate the secondary maximum on the side of the weaker source.

7. When the beat-frequency is much more rapid than one per second, not only do the secondary maxima disappear but localization in the region of 180° phase difference also vanishes.

These facts give a definite indication of the difference in origin of the primary and secondary maxima. But before we admit a complexity to account for this difference in origin, let us consider a physically simple equivalent of the skull and ear organs. Let us assume two physical instruments capable of recording vibrations and connected by numerous sound conducting paths. Is it possible for such an arrangement with two beating tones presented simultaneously one to each ear, to make a record that will show the three maxima and three minima actually experienced in each beat-cycle? Let us first assume that each instrument records intensity or a value proportional to the square of the amplitude at the instrument and that the resulting combined record is proportional to the arithmetical sum of these intensities. Upon this assumption we

[1] G. W. Stewart, loc. cit.

have already found, equation (3), that we may expect only a maximum at $\epsilon = 0°$ and a minimum at $\epsilon = 180°$.

If one assumes that the ear is an instrument that records not intensity but the time integral of the absolute value of the displacement, or $\int_0^{2\pi} |\text{displacement}|\, d\theta$,[1] he finds that the record in the nearer ear is proportional to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\cos(\epsilon - \epsilon_1 + \epsilon_2)}|$$

and in the farther ear to

$$|\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta\cos(\epsilon + \epsilon_1 - \epsilon_2)}|.$$

Reference to the equations just preceding (1) will show that these values are, in fact, the absolute values of the amplitudes. The total record, or the blended sensation, would be proportional to the sum of these two absolute values. It can be shown that if $\cos(\epsilon_2 - \epsilon_1)$ is greater than $2\alpha\beta/(\alpha^2 + \beta^2)$ this sum will be a maximum at 0° and a minimum at 180°. Obviously, $\cos(\epsilon_2 - \epsilon_1)$ is nearly unity, whereas the term $2\alpha\beta/(\alpha^2 + \beta^2)$ is, in all likelihood, much less than unity. We conclude that the foregoing assumption as to the dependence of the sensation upon the absolute value of the amplitude at the physical instrument leads to the same maximum and minimum as that based upon the dependence of the sensation upon the intensity of vibration. This leads one to doubt whether any assumption can be made as to the record of the instrument, the ear, which will result in a total sensation that has three maxima and three minima in one beat-cycle. At any rate, lacking such an assumption, the conclusion is that we must consider the ear not a single instrument, even for one tone. If, for our present purpose, one can consider each ear to consist of two different physical instruments which act somewhat independently of one another, a distinct gain is made thereby. For, assuming the sensations to be non-interfering, it is obvious that the sensations possible with two somewhat independent instruments are very different than those possible with a single instrument. For, suppose that instrument number one vibrates with the resulting vibration $f(t)$ giving the appropriate sensation, and that instrument number two vibrates with the resulting vibration $F(t)$, giving its appropriate sensation. If the sensations are non-interfering, the total sensation cannot, in general, be the same as had the vibrations $f(t)$ and $F(t)$ been combined in a single instrument.

Our theory will assume the equivalent of two physical instruments

[1] The writer is indebted for the suggestion of this assumption to Professor R. P. Baker, of the Department of Mathematics in the State University of Iowa.

in each ear excited in a mechanically different manner. A pair of like instruments, one on each side, is responsible for the primary maxima and minima, and another pair for the secondary maxima and minima. The former pair is excited chiefly through the ear drum-skin. The latter is excited chiefly through the vibrations of the skull. We will assume the mechanical arrangement to be such that the vibration conveyed by the skull from one fork to the distant ear has its phase determined not only by the equivalent length of path, but also by a change in phase of 180°. We will first discuss the consequence of the foregoing assumptions, and then its agreement with experiments.

The pair of A instruments, assumed excited chiefly through the membrana tympani, will give the primary maxima and minima provided we can also assume a conduction from one ear to the other. This conduction can occur by many paths and has been demonstrated actually to exist,[1] but the methods used could not exhibit the excellence of the conduction. We will assume this conduction as a fact without attempting to describe it in detail. Thus our previous theory, equation (3), can be regarded as applicable to the A instruments, and the primary maxima are completely explained.

The intensities in the B instruments can best be understood by reverting to the equations from which (1) and (2) were obtained. We will use α_1, β_1, E_1', E_2', ϵ_2' and ϵ_1' instead of α, β, E_1, E_2, ϵ_2 and ϵ_1 respectively, inasmuch as the values for the B instruments are different. Remembering the assumption that ϵ_2 really becomes $\epsilon_2' + 180°$ we obtain the displacement at the nearer ear, $\alpha_1 \cos(\theta + \epsilon - \epsilon_1') + \beta_1 \cos(\theta - \epsilon_2' - 180°)$ and at the farther ear, $\alpha_1 \cos(\theta - \epsilon_1') + \beta_1 \cos(\theta + \epsilon - \epsilon_2' - 180°)$. The intensities at the nearer and farther ears become, substituting λ' for $\epsilon_2' - \epsilon_1'$,

$$E_1' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon + \lambda') \tag{4}$$

$$E_2' = \alpha_1^2 + \beta_1^2 - 2\alpha_1\beta_1 \cos(\epsilon - \lambda'). \tag{5}$$

Taking the sum of intensities as before, we have,

$$E_1^2 + E_2^2 = 2(\alpha_1^2 + \beta_1^2) - 4\alpha_1\beta_1 \cos\epsilon \cos\lambda' \tag{6}$$

This sum has a maximum at 180° and a minimum at 0°. If the total sensation varies as the sum of equations (3) and (6), or the total intensity sum of both pairs of instruments A and B, then, no matter what values are assigned to our constants, we could obtain only one maximum and one minimum for one beat-cycle. This can be shown by the usual method of determining maxima and minima. But the assumption that

[1] K. L. Schaefer, Pflüg. Arch., Bd. 61, p. 544, 1895, and also C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 1908, A 80, p. 260.

insturments A and B are differently excited, leads us to assume also a difference in sensation from these excitations. If α_1 and β_1 have nearly the same value the intensity sum of the B instruments will be a maximum at 180° and a minimum, almost zero, at 0° phase differences. We would expect, then, the resulting observed effects at 0° to be chiefly the intensity-sum produced by the A instrument and at 180° to be the combination of the minimum of the A-intensity-sum and the somewhat different maximum of the B-intensity-sum. This is precisely the impression that an observer first obtains. He does not find two secondary maxima but only one in the neighborhood of 180° phase difference. Further, this maximum at 180° as already stated, appears to come from a source located in or on the skull, whereas the maximum at 0° is externally localized. This effect is in entire agreement with the assumption that the B instruments are excited by a skull vibration and the A instruments by an aërial vibration in the external meatus. The fact that the 180° maximum does not appear unless the beat-period is longer than two to five seconds indicates that the excitation of the B instruments requires a longer time, or that the inertia of the system is large, and this is in harmony with the assumption just mentioned.

But how account for two secondary maxima? In discussing the addition of intensities with the A instruments, we tacitly assumed a blending of the sound so that the maximum in either instrument was not noticed. This blending is, in fact, a part of our usual experience. But we have no reason for believing that the usual degree of blending must occur with the intensities at the B instruments. Indeed, the cause of blending (with the A instruments) is not understood. We may, therefore, without making any additional assumptions investigate what the effect would be if the observer is able to recognize the maximum intensity in each B instrument. Equations (4) and (5) show that the maximum intensity in the B instrument located on the side nearer the faster fork occurs before the maximum in the other B instrument by a phase of $2\lambda'$. Therefore, the maximum at the nearer B instrument would occur at $180° - \lambda'$ and at the farther B instrument at $180° + \lambda'$ phase differences. Further, the localization, because the excitation is from the skull, would appear to be first in the nearer and then in the farther ear. Again the theory is in distinct harmony with experiment. Thus, by assuming a difference between the A and B instruments in the manner of excitation and a difference in the degree of blending of the sensations, we can account for the appearance of the secondary maxima.

The difference in localization in the two parts of the beat-cycle, near 0° and near 180° phase difference, will be discussed in detail on a succeed-

ing page, but the comment may here be made that the cause is to be found in the difference in the manner of excitation of the two pairs of instruments.

The difference in the roughness of the sound in the two parts of the beat-cycle may also be explained by differences in excitation, for the excitation via the membrana tympani does, as we know, enable us to detect purity of tone, whereas the excitation directly by the skull would undoubtedly include natural vibrations of the skull itself and thus produce a somewhat rougher sound.

The rapid movement of the localization will also be presently discussed and satisfactorily explained on the basis of our theory.

The fact that δ (or λ' in our theory) is independent of the beat-frequency meets ready explanation for λ' represents the difference in retardation due to difference in the paths to the nearer and farther B ear, and certainly this value depends only upon the frequency of vibration and the conducting paths and not upon the rate at which the phase differences of the sources are changing. Or, in other words, λ' is entirely independent of ϵ and $d\epsilon/dt$.

But how does theory agree with the variation in $2\lambda'$ (our earlier 2δ) with frequency, as shown in Fig. 1 of a previous article?[1] We have assumed a retardation in phase of ϵ_1' in the propagation from the source

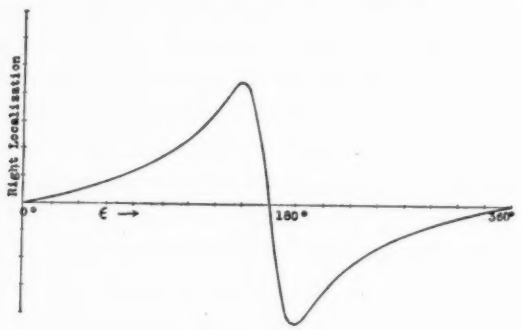

Fig. 1.

to the nearer instrument, and a retardation of ϵ_2' in the path from the source to the farther instrument. The excess retardation in conduction to the farther ear is thus $\epsilon_2' - \epsilon_1'$ or λ'. We can now introduce a helpful simplification without sensible error. Assume ϵ_1' occurs in a single path. Then it will be proportional to the frequency. But ϵ_2' will be a resultant of conduction over an indefinite number of paths, with varying amplitudes. Can ϵ_2' be regarded as occurring in an equivalent single path

[1] G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 507–513.

and hence be considered as proportional to frequency? As we will see, the answer is a negative one. For let us consider two paths only, along which the vibrations have the amplitudes a_1 and a_2. The difference in time required to transverse these paths is constant and hence the difference in phase at the distant junction of the paths is proportional to the frequency. Let this phase difference be γ, let r be the resultant amplitude, and let ϵ' be the phase difference between the resultant vibration and the one proceeding along the longer path, say, a_2.

Then

$$r^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \gamma \tag{7}$$

and

$$r \sin \epsilon' = a_1 \sin \gamma \tag{8}$$

From these equations we get, in case $a_1 = a_2$,

$$2 \cos \tfrac{1}{2}\gamma \sin \epsilon' = \sin \gamma$$

or

$$\epsilon' = \gamma/2.$$

Thus with equal amplitudes, or with $a_2 = 0$, ϵ' is proportional to γ and therefore to the frequency. The assumption of equality of amplitudes would therefore give us a change in the phase, ϵ_2', that is proportional to the frequency. But the assumption of equality of amplitude is not correct, for different paths would, on account of the differences in length and the presence of damping, have different amplitudes. We are then compelled to reject the suggestion that the various paths can be equivalent to a single path. Our expectation would therefore be that λ' or $\epsilon_2' - \epsilon_1'$ would change either more rapidly or less rapidly than in proportion to the frequency. Our experiments, as already described, show the latter to be the case.

As already stated, if the intensities of the two sources are far from equal, the general effect of a maximum in the neighborhood of 180° phase difference is retained, but the secondary maximum on the side of the less intensity is very noticeably lessened. This is apparently caused by the fact that there is enough blending of tone to cover up the individual maximum which is, as experiment shows, only recognized at all with practice.

We have thus discussed all the phenomena (excepting those presently to be considered under "localization") and have found that all are distinctly in harmony with the theory herein presented.

THE LOCALIZATION.

Lord Rayleigh[1] is inclined to favor perception of phase difference as one of the explanations of the phenomena of localization. At zero-phase

[1] Lord Rayleigh, Proc. Roy. Soc., 1909, A 83, pp. 61–64.

difference the sound is localized in front from which point it seems to wander about the head in the direction of the side of the tone of higher frequency, then into the ear. It then seems to pass suddenly (at 180° phase difference) through the head, reappearing in the other ear, and then back to the median plane in front by a path similar to that of the first half of the beat-cycle. C. S. Myers and H. A. Wilson[1] from experimental evidence showing the variation of the displacement of the localization with changing phase, accept the view that the lateral displacement is proportional to the difference of the intensities at the ears. If we subtract (2) from (1) we get $-4\alpha\beta \sin\epsilon \sin\lambda$. If now we assume a phase change of 180° to be introduced in transmission we get, by substituting $\lambda + 180°$ for λ, $+4\alpha\beta \sin\epsilon \sin\lambda$. The lateral displacement is then proportional to $\sin\epsilon$ and is zero when $\epsilon = 0°$ or 180° and is a maximum for $\epsilon = 90°$. This is the Myers-Wilson theory of localization. It does offer an explanation of the wandering of the sound toward and away from the median plane, but it is objectionable because by using this physical theory one cannot explain the primary maxima, the secondary maxima, the entrance of latter only when the beat-period is sufficiently long, the external localization in the region, $\epsilon = 0°$, the internal localization in the region $\epsilon = 180°$, the very small effect of excessive inequalities in intensities upon the localization in the region $\epsilon = 0°$ and the marked effect in the region $\epsilon = 180°$, the difference in rapidity of movement in the localization at $\epsilon = 0°$ and $\epsilon = 180°$, and the simultaneous disappearance of localization and secondary beats in the region $\epsilon = 180°$, when the primary beats are much more rapid than one per second.[2] In fact, the localization in the half cycle $\epsilon = 270°$ to $2\pi + 90°$ differs so much from the localization in the other half that one can hardly accept a theory that does not also make a distinction. In justice to Messrs. Myers and Wilson the statement should be made that their form of apparatus did not lead them to perceive most of the features that have been mentioned.

In the view of the writer the explanation of the localization is not so important as that of the primary and secondary maxima, for while the localization does not occur with all observers even with considerable practice, the presence of the maxima seems common to all. From the physicist's viewpoint, our theory would be that in the phase region, $270° < \epsilon < 2\pi + 90°$, difference of phase at the A instruments[3] is a

[1] C. S. Myers and H. A. Wilson, Proc. Roy. Soc., 1908, A 80, p. 260.

[2] Results given by G. W. Stewart, PHYS. REV., 2d Series, Vol. IX., No. 6, 1917, pp. 502–508.

[3] For our present purpose it maybe sufficient to note that the phase of the fork nearer to an ear will furnish the greater portion of the amplitude and thus that the variation in phase difference at the A instruments is largely determined by the phase difference of the forks themselves.

determining factor, and that in the region $90° < \epsilon < 270°$ the non-blending excitation of the B instruments, first with greater intensity on the side of the higher pitch, and then with equal intensity and then with greater intensity on the other side, explains the localization. The explanation of the region $270° < \epsilon < 2\pi + 90°$ by phase difference will not meet the approval of many psychologists, on the ground that sensation depends upon the mode of response of the end organ and not upon the character of the stimulus. But one must not treat too seriously the application of a generalization that has never been definitely established. Moreover, inasmuch as all observers do not obtain the variation of localization herein described, we may assume it to be a "second-order effect" or one whose cause may never have been detected in experiments with sensory stimuli. Peterson[1] considers the localization in this region as caused by phase differences and concludes that the perception of phase difference is cortical in origin. The explanation given above of the localization in the region $90° < \epsilon < 270°$ does not meet with the objection just cited or any other of which the author is aware. Moreover, it should be anticipated that the excitation of the A instruments by the usual route, membrana tympani, etc., would give an external localization and the excitation of the B instruments, directly by the skull, an internal localization, just as is always produced by any vibration given the skull only. The effects produced by unequal intensities at the ears also accord with the theory. For if the localization near $\epsilon = 0°$ is due chiefly to a perception of phase difference by the A instruments the effect of unequal intensity should be slight. Again, if the localization near $\epsilon = 180°$ is caused by the lack of complete blending of the intensities at the B instruments, it would be seriously modified by inequality in the intensities at the ears. Both of these conclusions are verified by experiment. The rapid movement of the localization in the region of 180° is in accord with the theory, which gives the variation in phase difference with localization in the ears as $2\lambda'$ or $2(\epsilon_2' - \epsilon_1')$, for, from physical considerations, we can see that $\epsilon_2' - \epsilon_1'$ must be a small angle. Suppose the equivalent difference of path in the skull bones were 20 cm. If this were in air, the phase introduced thereby for a frequency of 256 d.v. would be about 60°. But the value λ' or δ for this frequency is approximately 45°.[2] Inasmuch as the velocity in the skull bones must be several times[3] that in the air, we find that the theory would suggest an angle even smaller than that found by experiment. Both theory and experiment agree qualitatively that the dif-

[1] J. Peterson, *loc. cit.*

[2] G. W. Stewart, PHYS. REV.

[3] H. Frey, Ztschr. f. Psych. u. Phys. d. Sinnesorgane, Bd. 28, p. 9, 1902, gives *ten times*.

ference between the phase relations at the two secondary maxima is markedly less than 180° and hence that the passage from the first secondary maximum (following 0° phase difference) to the other would seem to be more rapid than the movement in the remainder of the beat-cycle. The quantitative agreement is however, not satisfactory.

The theory is further confirmed by the fact that with a rapid beat-frequency (more rapid than one per second) not only do the secondary maxima disappear, but the localization in the 180°-phase-difference region disappears also.

ANATOMICAL CONSIDERATIONS.

The query arises, does the structure of the ear permit of the assumptions as made? It would seem that no serious objection arises if we assume the instrument A to be the organs of Corti in the ductus cochlearis, and the instrument B to be the similar organs in the saccule or the utricle. These organs in the cochlea and the saccule and utricle, have a common origin. Herrick[1] states that some physiologists have thought that cochlear and vestibular nerve systems are not wholly distinct and that the sense organs in the saccule may also function as a sound perceptor. This uncertainty concerning the function of the organs in the utricle and saccule on the part of the physiologists does not serve as an objection to our theory. Indeed, the fact that the complex phenomena herein described can be accounted for by the assumption of the existence of the B instruments ought to assist in deciding this uncertainty. The fact that there is a great difference in the central connections of the cochlear and vestibular nerves is in harmony with our contention that the instruments B give sensations which do not completely blend and which do not produce external localization.

But are the vestibular and cochlear organs mechanically different? Obviously they are, for the latter are placed upon a membrane, and are apparently excited by the motions of the fluid in the scala media, of the basilar membrane and of the tectorial membrane. The vestibular organs are located at the points where the membranous labyrinth is fastened to the skull itself. Moreover, the movement of the vestibular fluid must be very small for the vestibular canals are closed. Thus there would seem to be a mechanical difference which corresponds admirably with our assumption that instrument A is excited via the vibration of the membrana tympani and that instrument B is excited by the skull. The only change in our assumption induced by the structure of the ear would

[1] C. J. Herrick, Introduction to Neurology, pp. 201–202.

be the possible insertion of the word "chiefly" after each word "excited" occurring in the preceding sentence.[1]

Restatement of Theory and Agreement with Experiment.

By the assumption of two physical instruments A and B on each side, having a difference in mechanical arrangement, a difference in the method of excitation, a difference in the blending of the sensations of the A pair and B pair of instruments and an independence of sensations produced by the two pairs and the additional assumptions that instruments A, the usual instruments of hearing, can perceive phase differences, and that in the skull conduction to the B instruments there is an additional phase change of 180°, we have a theory that explains fairly satisfactorily the presence of the primary maxima, the secondary maxima, the appreciable length of beat-period requisite for the appearance of the latter, the localization in the entire beat-cycle, the difference in the nature of the localization and in the quality of the sound in the two halves of the beat-cycle, the rapid movement of localization between the secondary maxima, the variation of the separation of the secondary maxima with the fork-frequency and its independence of beat-frequency, the difference in the effects of unequal intensities upon the localization in the two halves of the beat-cycle, and finally the simultaneous disappearance of the secondary maxima and the localization in the region of 180°-phase-difference.

Our theory does not lead us to identify the B instruments, but merely to suggest the possibility of these instruments being located in the utricule and saccule. Anatomical and physiological considerations give evidence in favor of the suggestions.

The theory is, of course, incomplete, but it has herein proved to be very successful and hence promising. There are no conspicuous phenomena connected with binaural beats that are not in accord with the theory, but there are minor points that should receive further consideration. For example, S. P. Thompson[2] has found binaural beats occurring when the tones used have frequencies of almost two to one.

While it is known[3] that the secondary maxima and the localization phenomena occur both when the sounds are presented by tubes which

[1] A paper of importance has just come to the writer's notice. D. Richards (Zeitsch. f. Biol., 1916, 66, 579–609) reports upon experiments with a guinea pig in which there were removed (1) both cochlea, (2) both cochlea and one vestibule and (3) both labyrinths. His conclusion is that sound stimuli may be regarded as adequate stimuli for the vestibular apparatus, but that no conclusion can be drawn as to any sensation produced in this way. This contribution is entirely favorable to the theory presented in this paper.

[2] S. R. Thompson, Phil. Mag., (5), IV, 1877, p. 274.

[3] G. W. Stewart, Phys. Rev., 2d Series, Vol. IX. No. 6, 1917, pp. 502–508.

approach but do not close the opening of the external meatus, yet there may be a difference in the quantitative measurement of 2δ with the ear open and closed. The experiments of the writer have been made almost entirely with binaurals that close the external meatus, and accurately speaking his theory has reference to that experimental condition. Nevertheless he believes it to be equally applicable to the open-ear-binaurals.

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ON THE EFFECT OF DISTRIBUTED CAPACITY IN SINGLE LAYER SOLENOIDS.

BY J. C. HUBBARD.

THE present paper describes a precise method of measuring the periods of electric oscillations, from the results of which is determined the effective, or distributed capacity of coils of a single layer over a wide range of coil dimensions. The method is checked by a determination, using the measured periods and the calculated inductances and external capacities, of the quantity v, the ratio of the electromagnetic to the electrostatic unit of quantity.

The problem of a coil oscillating in its own free period has proved one of the most difficult in mathematical physics. In addition to its inductance the coil possesses an effective capacity resulting from the fact that different parts of the coil are at different periodically varying potentials. There has been no theory developed except for coils of a single layer, and the theory which has been so developed agrees only roughly and within restricted limits with the experimental data. Drude[1] developed expressions for the wave-length and capacity of a freely oscillating coil and measured the wave-lengths of coils over a wide range of coil dimensions. A more rigorous treatment has been given by W. Lenz[2] who developed expressions for the wave-lengths of extremely long and of extremely short coils and covered the region between by interpolation.

Another method of finding the free periods consists in measuring the period of a circuit composed of an inductance and an external capacity in series. The square of the period should be a linear function of the external capacity. Thus, $T^2 = 4\pi^2 L(K + K_0)$, where K is the capacity of the external variable condenser, and K_0 is the remaining capacity of the system, composed in part of the distributed capacity of the coil. K_0 may thus be obtained by extrapolation. Such measurements were made by Professor Webster[3] who verified the linear relation. Experi-

[1] P. Drude, Ann. d. Phys., (4), 9, pp. 293–339, 1902.

[2] W. Lenz, Ann. d. Phys., (4), 43, pp. 749–797, 1914. See also, K. W. Wagner, Das Eindringen einer elektromagnetischen Welle in eine Spule mit Windungskapazität, Elektrotechnik u. Maschinenbau, Wien, Nos. 8 and 9, 1915; Die Theorie des Kettenleiters nebst Anwendungen, Archiv für Elektrotechnik, Nos. 10 and 11, 1915.

[3] A. G. Webster, An Experimental Determination of the Period of Electrical Oscillations, PHYS. REV., VI., pp. 297–314, 1898.

ments along this line were projected by the writer in 1911, making use of a new method for measuring periods, with the special object of determining with accuracy the effective capacity of the coils used. Results were presented before the American Physical Society in October, 1912, and in April, 1915."[1]

In the meantime Professor G. W. O. Howe[2] in a paper on the calibration of wave-meters had shown the linear relation to hold down to the free period of the coil and that the effect of the self-capacity of the coil is to add a constant capacity to that of the system.

W. Rietz,[3] making use of a Helmholtz pendulum traced the wave form of an oscillating circuit with variable capacity and came to the same conclusion. His results, for the one single-layer coil studied, give only the order of magnitude of the distributed capacity. Frederick A. Kolster[4] has shown how the distributed capacity of a coil may be found from the apparent inductances at various frequencies.

The present paper is an attempt to supplement the work of Drude, each coil being part of an oscillating circuit of the sort common in wireless telegraphy and instruments, one end of the coil being earthed, and not, as in Drude's experiments, oscillating freely as in the Tesla transformer.

MEASUREMENT OF PERIOD.

The method here used for measuring periods has already been briefly described by the writer.[5] It makes use of a modification of the apparatus used earlier by Professor Webster and owes its precision largely to the excellent design of his drop chronograph. The modification introduced by the writer consists of an arrangement by means of which an electrometer is made to give deflections of a sign and magnitude depending upon the potential at any desired instant after the interruption of the current in an inductive circuit. If the circuit be oscillating the device enables the oscillations of potential to be traced and furnishes a very precise means of measuring the period of oscillations.

Lever *1* (Fig. 1) closes the circuit through the inductance L, the amount of current being regulated by the non-inductive resistance R. The condenser K is connected across the inductance terminals. A weight is released by an electromagnet and falls freely through a definite vertical distance striking lever *1* and opening the circuit. Oscillations ensue, the period of which depends upon the system K, L and connections, the

[1] J. C. Hubbard, PHYS. REV., N.S., I., pp. 247–249, 1913, VI., p. 58, 1915.
[2] G. W. O. Howe, Proc. Phys. Soc. Lond., XXIV., pp. 251–259, 1912.
[3] W. Rietz, Ann. d. Phys., (4), 41, pp. 543–569, 1913.
[4] F. A. Kolster, Proc. Inst. Radio Eng., I., No. 2, pp. 19–34, 1913.
[5] *Loc. cit.*

variable resistance and battery being cut out and earth-connected at E. At a definite moment later the projectile strikes lever *2* which, in turn strikes lever *3*, lifting it from the contact P. Lever *2* is permanently connected with one plate of the air condenser and may be regarded as a potential terminal. The contact P is connected with one pair of electrometer quadrants which are disconnected from earth at the moment of release of projectile. As may be seen, connection is established and immediately broken between the electrometer at the moment that lever *2* strikes *3*. The phase of oscillation at this moment determines the sign and magnitude of the charge received by the electrometer. The duration of the time of contact between condenser and electrometer must be considered as finite, but the results show that it is either extraordinarily constant or very small compared to the periods used.

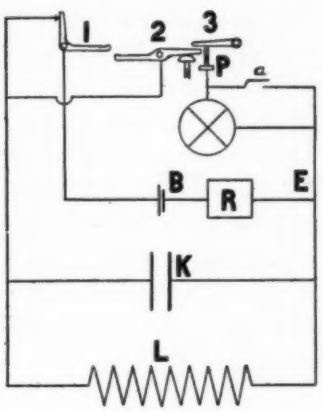

Fig. 1.

The system comprising levers *2* and *3* and the contact screw P, in addition to a catch (not shown) to prevent lever *3* from returning to P during the electrometer deflection, is mounted on a heavy plate of ebonite which is screwed to a carriage moved vertically by a micrometer screw reading to 0.001 mm. In practice it is customary to determine frequently the highest position for the carriage at which deflections begin to be observed in the electrometer. This is the point at which contact is made with the electrometer at the same instant as the breaking of the circuit and is taken as the micrometer zero. At this point the projectile has a velocity depending upon its height of fall above lever *1*. Subsequent readings of the micrometer, referred to this point as origin, thus measure the further excursion of the projectile, from which its average velocity during any interval that oscillations are taking place, and hence the time required, may be found. In making the determination of a period it is customary to locate positions for two successive nodes, then to look for a node at twice this distance, and so on, doubling each time, until a suitable number of nodes, sometimes several hundred, have been covered. The initial and final nodal positions of the screw are determined to 0.0001 cm., and a range of not less than 0.5 cm. is covered, giving an accuracy in the length measurement better than one part in two thousand. The period of oscillation is then $T = s/nV$, where s is the distance on the micrometer screw corresponding to $2n$

half waves, and V is the average velocity of projectile during the oscillations which are being measured. It appeared that there were four possible sources of error in the determination of V, each tending to make the actual value less than that found; viz., residual magnetism in the electromagnetic release of the projectile, air resistance, moment of inertia of the levers, and tension of the rubber bands holding the levers in place. By using various values of magnetization in the electromagnet, various velocities up to 325 cm./sec., with consequent variation of air resistance, by trying levers of different dimensions and under different degrees of tension, it is found that these errors are not of a magnitude, singly or collectively, to be detected in the results.

The falling weight, or projectile, is that of Professor Webster's drop chronograph, a description of which has been published.[1]

THE ELECTRICAL SYSTEM.

The solenoid, the distributed capacity of which is to be measured, is mounted, axis horizontal, at a distance of several feet from the condenser. The leads from the solenoid to the condenser form a rectangular circuit of known dimensions (Fig. 2), the plane of which passes through the axis

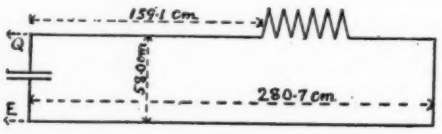

Fig. 2.

of the coil. The upper side of the rectangle is a continuation of the generating line of the coil surface on its lower side, the wire of the coil at the ends making a right angle turn about a peg set into the core. The wire Q, leads to contact with levers *1* and *2*.

The air condenser consists of two plates of ground and polished steel held apart by sets of three small glass cylinders of length determined by an optical micrometer. Six sets of cylinders furnish the capacities (Kirchhoff's formula) given in Table III.

Tables I. and II. give the dimensions and calculated values of inductance of the accurately constructed coils. Rosa has given tables from which Nagaoka's formula for the inductance of a current sheet may be corrected to represent the actual inductance of a coil. The following data are necessary:[2] r = mean radius of coil, b = length from center to center of $n + 1$ turns $= (h - d')n/(n - 1)$, where h = axial length of

[1] *Loc. cit.*

[2] E. B. Rosa, Bull. Bu. Standards, 8, No. 1, p. 119, 1912–13.

the coil from outside to outside of insulated wire of diameter d'. d = diameter of bare wire, D = pitch of winding = $(h - d')/(n - 1)$. The inductance of the rectangular circuit is calculated from Rosa's formula,[1] in which a = length, b = breadth of rectangle, and ρ = radius of bare

TABLE I

Data for Coils.

Series.	r cm.	d cm.	d' cm.	D/d	Series.	r cm.	d cm.	d' cm.	D/d
E	9.740	.0650	.0840	1.50	K	6.297	.0650	.0840	1.40
F	21.163	.1291	.1529	1.63	L	31.57	.0636	.0844	4.98
G	9.84	.0625	.0834	3.13	M	9.64	.0627	.0840	5.06
H	31.63	.1810	.2095	1.75	N	9.745	.0627	.0840	1.56
I	31.55	.0632	.0830	5.02	O	21.165	.0650	.0860	3.26
J	7.96	.0659	.0855	1.35					

TABLE II.

Dimensions and Inductances of Coils.

Series.	n.	h.	$L' \times 10^{-6}$ Cm.	Series.	n.	h.	$L' \times 10^{-6}$ Cm.
E_1	886	86.549	30.904	H_3	70	22.06	3.839
E_2	750	73.263	25.732	H_4	50	15.75	2.273
E_3	625	61.050	20.993	H_5	35	10.95	1.283
E_4	500	48.837	16.279	I_1	162	51.19	12.926
E_5	375	36.624	11.609	I_2	100	31.50	6.551
E_6	250	24.412	7.0403	I_3	70	21.97	3.850
F_1	215	45.37	12.637	I_4	50	15.58	2.288
F_2	180	37.97	9.993	I_5	35	10.84	1.292
F_3	145	30.58	7.442	J	216	19.2	4.41
F_4	110	23.18	5.035	K	619	56.20	9.694
F_5	85	17.90	3.448	L	162	51.17	12.946
F_6	60	12.61	2.027	M	264	83.61	2.796
G_1	436	85.10	7.763	N	868	84.88	30.202
G_2	300	58.51	5.126	O_1	215	45.33	12.674
H_1	161	50.92	12.861	O_2	180	37.94	10.021
H_2	100	31.61	6.539	O_3	145	30.54	7.465

wire. The wire of the coils of series E to O was double cotton covered. Coils E to I and L to O, inclusive, were wound on built up cores of white pine with the grain parallel to the axis of the coil. Coil K was wound on a tube of ebonite of 12.5 cm. external, and 10.1 cm. internal diameter, and of length, 60.5 cm. All cores except that of series J were given a light thread in the lathe barely sufficient to hold the tightly wound wire in place. Coil J was closely wound on a glass cylinder of thin walls (2 mm.).

The total inductance, L, of the system is obtained by adding the induc-

[1] *Loc. cit.*, p. 155.

tance of the coil, L', given in Table II., to that of the rectangular circuit, L''. This would, of course, be strictly true only if the terminals of the coils had been led to the axis before coming off, but the change is so slight as not to appear in any figures retained for total inductance.

Preliminary experiments were conducted to test the effect of changing the dimensions of the rectangular lead circuit and the relative position of coil and condenser. There was no sensible difference in the value of K_D or of v obtained with the same coil in the three circuits used. The dimensions of the circuit used in the final experiments are given in Fig. 2, ρ being 0.0251 cm., and the inductance of the circuit, $L'' = 10{,}260$ cm.

RESULTS.

The period of oscillation of the electrical system is, by reason of the small resistance in the oscillating circuit and the relatively small capacity, represented by $T = (2\pi/v)\sqrt{LC}$, where L is the combined inductance of coil and rectangular circuit, and C is the combined capacity of the system, consisting of the air condenser K, the capacity of leads and connections K_L, and the distributed capacity of the coil, K_D. Thus,

$$T^2 = \frac{4\pi^2 L}{v^2}(K + K_L + K_D).$$

If there is no appreciable variation of L or of K_D with the frequency, T^2 will be a linear function of K, or $T^2 = \alpha K + \beta$, where $\alpha = 4\pi^2 L/v^2$, and $\beta = (4\pi^2 L/v^2)(K_L + K_D)$. The constants α and β may be determined by the method of least squares, giving

$$\beta/\alpha = K_L + K_D = K_0, \quad \text{and} \quad v = 2\pi\sqrt{L/\alpha}.$$

Table III. is chosen to illustrate the manner in which these quantities are determined, and is an example of the best work done with the drop chronograph, though a large majority of the thirty-four similar tables

TABLE III.

Determination of α and β: Series E_2.

K Cm.	$T^2 \times 10^{10}$, Obs.	$T^2 \times 10$, Calc.	$\Delta T^2 \times 10^{10}$.	$\Delta T^2/T^2$.
211.32	2.7778	2.7779	−.0001	−.00004
332.04	4.1472	4.1464	+ 8	+ 19
427.47	5.2261	5.2282	− 21	− 40
565.05	6.7912	6.7880	+ 32	+ 37
827.56	9.7587	9.7636	− 49	− 50
1,528.71	17.7164	17.7119	+ 45	+ 25

$\alpha = .011336 \times 10^{-10}$ $\quad L = 25.742 \times 10^6$ cm.

$\beta = .38237 \times 10^{-10}$ $\quad v = 2\pi\sqrt{L/\alpha} = 2.9943 \times 10^{10}$ cm./sec.

$K_0 = \beta/\alpha = 33.73$ cm.

at hand show an accuracy as great, and in only two cases, owing to trouble with the drop chronograph, did the average value of $\Delta T^2/T^2$ exceed one tenth per cent. Table III. illustrates very effectively that, to a high degree of precision, there is neither a variation of inductance or of distributed capacity in the coils as here used.

Having determined K_0 as in Table III., it is next necessary to determine K_L, the capacity of leads, in order that the distributed capacity, K_D, of the coil may be found. The capacity of leads has been directly measured by an electrostatic method.[1] This is not a satisfactory procedure, as it is necessary in making such a measurement to disconnect the condenser and coil from the lead wires, tending to raise somewhat the capacity of the leads. A more satisfactory method consists in finding values of K_0 for geometrically similar coils of different linear dimensions. Suppose one coil is of linear dimensions n greater than another. According to the well-known theorem of electrically similar systems the distributed capacity of the larger coil should be n times that of the smaller. Thus $n(K_D)_1 = (K_D)_n$, whence $(\beta/\alpha)_1 = K_L + (K_D)_1$, and $(\beta/\alpha)_n = K_L + n(K_D)_1$, from which an independent determination of K_L and K_D is possible. If we consider coils of the same core material and of the same ratio of pitch to diameter of bare wire, we may put, after Drude, $K_D = r\phi(h/2r)$, whence $K_0 = \beta/\alpha = K_L + r\phi(h/2r)$. From graphs of K_0 as a function of $h/2r$ (see Tables I., II. and IV.) for the series E, F and H, which are strictly comparable, we may determine K_0 for the corresponding values of $h/2r$. For $h/2r = 1$, for example, we have,

$$\text{E}, \quad K_0 = 32.15 = K_L + 9.74\phi,$$

$$\text{F}, \quad K_0 = 44.85 = K_L + 21.16\phi,$$

$$\text{H}, \quad K_0 = 54.52 = K_L + 31.63\phi,$$

from which $K_L = 22.25$ cm. (See inserted figure in upper right hand corner of Fig. 4.) The value measured directly by the electrostatic method is 23.4 cm., an excellent agreement, being slightly larger, as is to be expected. We then have, $K_D = K_0 - 22.25$. The results are given in Table IV.

It is evident from the results that the factor of chief importance in determining the distributed capacity is the radius of the coil, in all cases the two quantities being of the same order. This agrees with Drude's theory, insofar as it concerns the radius alone, but the form of his expression for distributed capacity does not hold in the present case, the coils here being earthed at one end.

[1] J. C. Hubbard and H. F. Stimson, PHYS. REV., N.S., I., pp. 245–247, 1913.

The values of K_D/r from the results of this paper are plotted in Fig. 3 with corresponding values of $h/2r$. It will be observed that K_D/r is a minimum and very nearly equal to unity when $h/2r = 1$. Drude's

TABLE IV.

Series.	$\alpha \times 10^{12}$.	$\beta \times 10^{12}$.	K_D Cm.	Series.	$\alpha \times 10^{12}$.	$\beta \times 10^{12}$.	K_D Cm.
E_1	1.3614	46.964	12.24	H_3	.16892	9.3184	32.92
E_2	1.1336	38.237	11.48	H_4	.10028	5.4961	32.56
E_3	.92476	30.685	10.93	H_5	.056886	3.2120	32.92
E_4	.71627	23.973	11.22	I_1	.57062	30.756	31.65
E_5	.51132	16.666	10.34	I_2	.28938	15.735	32.10
E_6	.31106	9.998	9.89	I_3	.170015	9.3104	32.51
F_1	.55547	25.115	22.96	I_4	.101221	5.4000	30.81
F_2	.44066	19.455	22.00	I_5	.057251	3.0750	31.34
F_3	.32810	14.683	22.50	J	.19416	5.6855	7.03
F_4	.22219	10.154	23.45	K	.42832	12.915	7.90
F_5	.15228	7.0226	23.87	L	.57046	30.803	31.75
F_6	.089730	4.2956	25.62	M	.12342	4.348	13.48
G_1	.34434	11.874	12.23	N	1.3373	45.700	11.92
G_2	.22706	7.6549	11.46	O_1	.55899	25.445	23.30
H_1	.56543	30.836	32.29	O_2	.44284	20.215	23.40
H_2	.28816	15.737	32.36	O_3	.32973	15.108	23.57

theory, modified to suit the present case, and *assuming the total inductance to be effective*, gives values of distributed capacity in fair agreement (within ten per cent.) with results for the longest coils only. Further reference to Drude's work will be found below. It is interesting to

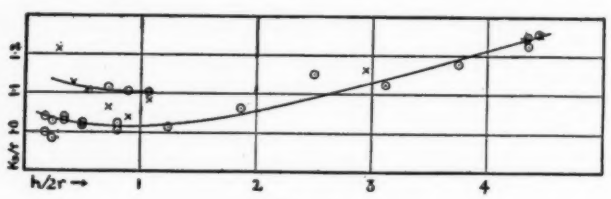

Fig. 3.

note that the increase in distributed capacity on the lower side of the minimum ($h/2r < 1$) is much less rapid than on the upper side. The results would seem to justify Lenz's mode of treatment, since we apparently have two quite independent factors, one predominating greatly in very short coils, the other, in very long coils.

There is no evidence that the variation of ratio of pitch to diameter of wire has a measurable effect on the distributed capacity in the region studied, though some effect is to be expected for coils of a smaller number of turns than those studied here. Such changes as here exist may

quite certainly be attributed to slight changes in the position of the wire leading from the condenser to the lever system.

The nature of the core in the present experiments seems to be of relatively small importance in the magnitude of distributed capacity. For example, the ebonite tube, Series K, gives, for $h/2r = 4.46$, a value of $K_D/r = 1.254$, while for coil E_1, solid wooden core, $h/2r = 4.44$ and $K_D/r = 1.257$, a difference well within such accidental errors as might occur. On the other hand, Series J, close wound glass cylinder, $h/2r = 1.205$, gives $K_D/r = .884$, a result twelve per cent. smaller than the corresponding value for wooden cores, but, on filling the cylinder with kerosene the value is raised by only one per cent.; in the cases of both the empty and filled cylinder the values lie much closer to those for corresponding coils on wooden cores than do Drude's results for freely oscillating coils on glass cylinders.

In the present experiments, accordingly, the dielectric nature of the core is less important, owing to the fact, as before noted, that one end of the coil is connected to earth, producing a different configuration of the field of electric force from that in Drude's experiments. Fewer lines, relatively, would traverse the core.

To bridge the gap between these experiments and those of Drude the periods of two of the larger coils, with a threefold variation of radius, were measured with $K = 0$, *i. e.*, with the air condenser disconnected. Since $T^2 = \alpha K + \beta$, we have $T_0 = \sqrt{\beta}$. The results follow.

Series L, $\alpha = .0057046 \times 10^{-10}$, $\sqrt{\beta} = .5554 \times 10^{-5}$, T_0 observed $= .57223 \times 10^{-5}$.

Series N, $\alpha = .013373 \times 10^{-10}$, $\sqrt{\beta} = .67602 \times 10^{-5}$, T_0 observed $= .70765 \times 10^{-5}$.

Let Δ be the small positive change introduced in the lead wire capacity by disconnecting the condenser. Then, $T^2 = \alpha\Delta + \beta$, and $\Delta = (T_0^2 - \beta)/\alpha$, giving $\Delta = 3.41$ cm. for Series L, and 3.28 cm. for Series N. Taking as the mean, $\Delta = 3.34$ cm. and calculating $T_0^2 = 3.34\alpha + \beta$, we have for calculated periods of L and N respectively, .5718 and .7082 $\times 10^{-5}$ sec., the corresponding observed periods being .5722 and .7076 $\times 10^{-5}$ sec.

This is a remarkably good agreement, and shows that the discrepancy between T_0 and $\sqrt{\beta}$ may entirely be explained by the change introduced in lead capacity. It may be concluded that, to a high degree of precision, the square of the period of a coil and condenser circuit is a linear function of the external capacity down to the value of the capacity of the coil.

This conclusion draws attention to a very interesting matter, also discussed in Howe's paper. In all our work we have assumed that the

total inductance of the coil is effective. Drude assumes a distribution of current in a freely oscillating coil such that the total inductance is multiplied by the factor $2/\pi$ to give the effective inductance. The effective capacity of the coil would then be $\pi/2$ times the values we have given. Since the relation $T^2 = \alpha K + \beta$, holds down to $K = 0$, with precision, it would seem that either the total inductance must be effective in a freely oscillating coil, or there must be some exactly compensating change in the effective capacity.

The free periods of all the coils used, earthed at one end, have been calculated by putting $K = -K_L$, giving $T_F = -\alpha K_L + \beta$, and $\frac{1}{2}\lambda_F = \frac{1}{2}\,3 \times 10^{10} T_F$. The results follow very closely Drude's expression $\frac{1}{2}\lambda = lf(h/2r, D/d, e)$ if we take for l *twice* the length of wire in the coil, and for f values in his tables corresponding to values of h equal to *twice* the axial length of coil for *coils without cores*. The values of f are plotted with corresponding values of $2h/2r = h'/2r$ in Fig. 4. (Series E and H,

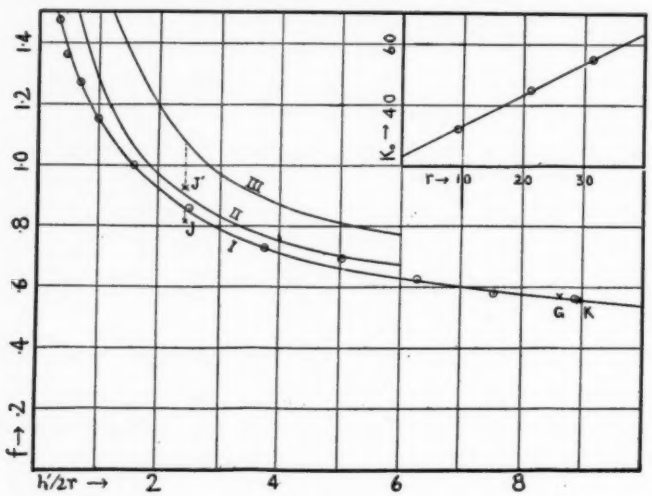

Fig. 4.

only, are plotted for the sake of clearness.) The smooth curve, I., drawn through these points, closely parallels Drude's curve, II., for similar coils without cores, in which $h' = h$. The curve, III., is drawn from Drude's results for corresponding coils on wooden cores, and, as may be seen, lies much farther away from the results of the present paper. The point K, giving the value of f for Series K (ebonite tube), is seen to correspond well with the results of this paper for wooden cores. From Drude's work a value considerably less would be expected. The departure of the value of f for series J (coil wound on glass cylinder), from the curve for wooden cores is seen to be much smaller than the

departure of Drude's corresponding value for a coil on a glass cylinder, J', from his results for similar coils on wooden cores. All these facts are in further support of the conclusion that, for coils earthed at one end the material of the core has much less effect on the magnitude of the distributed capacity than it has in coils freely oscillating with a potential node at the center.

MEASUREMENT OF v.

As a check on the accuracy of the drop chronograph, a calculation of v, the ratio of the electromagnetic to the electrostatic unit of quantity, has been made from the results. We have,

$$T^2 = (2\pi/v)^2 L(K + K_0) = \alpha K + \beta,$$

from which,

$$v = 2\pi\sqrt{L/\alpha}.$$

Table III. gives an example of v so computed. From the data already given v may be calculated in each case. It will be found that the values are in excellent agreement. Owing to the greater care taken in the construction and measurement of dimensions of the coils of Series E and F we shall use the values of v obtained in those series as examples. We have for the average deviations of r and h, respectively; for E, .003 and .003 cm.; for F, .008 and .005 cm. Table V. gives the values of v, the computed deviation in v, Δv_c, as well as the actual deviation, Δv_0, from the mean.

TABLE V.

Determination of v.

Series.	$v \times 10^{-10}$.	$\Delta v_c \times 10^{-10}$.	$\Delta v_0 \times 10^{-10}$.	Series.	$v \times 10^{-10}$.	$\Delta v_c \times 10^{-10}$.	$\Delta v_0 \times 10^{-10}$.
E_1	2.9942	±.0013	−.0008	F_1	2.9983	±.0013	+.0030
E_2	43	10	− 7	F_2	39	16	− 13
E_3	50	11	± 0	F_3	52	13	± 0
E_4	63	13	+ 13	F_4	43	16	− 9
E_5	52	11	+ 2	F_5	45	14	− 7
E_6	14	—	—	F_6	39	—	—

The values of v for E_6 and F_6 are rejected because the smallness of periods in these cases made it impossible to carry out measurements with the smaller values of external capacity, giving values of α of a lower order of precision than in the other cases. From the first five results in each series, we have,

E, $v = 2.9950 \times 10^{10}$ cm./sec.,

F, $v = 2.9952 \times 10^{10}$ cm./sec.,

with an accuracy better than one part in two thousand, a much higher degree of accuracy than has before been obtained by this method. This

is the more interesting because the most authoritative writers on the subject have expressed the opinion that an accuracy greater than one part in one thousand could not be reached by the present method.[1]

The best determination of v which has been made by any method is that by Rosa and Dorsey,[2] who obtained the value, $v = 2.9963 \times 10^{10}$ cm./sec., with an accuracy of one part in ten thousand.

The values of v here obtained are such as to commend the drop chronograph as an excellent instrument for the precise measurement of small intervals of time, for the calibration of wave-meters, and to establish the value of the present method of measuring periods. It would be possible, with small additional labor, to increase the accuracy of the time measurement to one part in ten thousand. This would give a value of v, using properly constructed coils, with a precision probably as great as that of Rosa and Dorsey.

The present method of measuring periods with the drop chronograph is especially useful for getting precise measures of small capacities or of variations of capacity, any capacity which has been introduced appearing as a change in K_0. In this way the capacities of lead wires, of electrometers, and the mutual effects of conductors have been studied. The apparatus as used in getting the results in this paper is sensitive to a change in K_0 of one part in one thousand, or, of .01 to .04 cm. of capacity. It is obvious that the method is also useful for the precise determination of inductance, and should be especially valuable, because the capacity of the coil is eliminated.

Dielectric constants of insulating liquids have been measured by the writer. For the purpose a small air condenser in a suitable cell was introduced in series with K and the period measured with it empty, or filled with the desired liquid. When conducting liquids are used the damping is so great that precise values of T cannot be obtained by this method. A resonance method has accordingly been developed for the study of dielectric constants of conducting liquids. This method, using a circuit calibrated for wave-length by the drop chronograph, and an audion detector, is capable of yielding good results also for distributed capacity of coils.

Some work has been done on the distributed capacity of coils of more than one layer. This will appear in a later paper.

In conclusion it may be remarked that information is greatly needed concerning the distribution of current and potential in oscillating coils. Until such information is at hand it is probable that the theoretical treatment of the single layer solenoid will not make much headway.

[1] E. B. Rosa and N. E. Dorsey, Bull. Bu. Standards, 3, p. 617, 1907.

[2] E. B. Rosa and N. E. Dorsey, Bull. Bu. Standards, 3, pp. 433–604, 1907.

SUMMARY.

The present paper describes a precise method of measuring the electrical periods of circuits, from the results of which are calculated the effective, or distributed capacities of coils earthed at one end. The work covers a wide range of coil dimensions.

1. It is found that the relation between the square of the period and the external capacity is very accurately linear down to the free period of the coil. This relation is new only in the accuracy with which it has been proved.

2. That the distributed capacity of coils earthed at one end is not much affected by the nature of the core.

3. The distributed capacity of such coils is a minimum and practically numerically equal to the radius of coils of length equal to the diameter, increasing, for example, by twenty-five per cent. when the length of coil is four times its diameter. For the practical purposes of wireless telegraphy it would seem sufficiently accurate to consider the effective capacity of the coil as equal to its radius in cm. of capacity, or to $(r/9) \times 10^{-5}$ microfarads.

4. The results, extrapolated to the natural periods of the coils, for coils earthed at one end, and with wooden cores, are in good general agreement with the results of Drude for freely oscillating coils *without* cores and of *twice* the axial length. This gives additional support to conclusion (2).

5. The method is checked by a determination from the results, of v, the ratio of the electromagnetic to the electrostatic unit of quantity. Values are found having an accuracy better than one part in two thousand, establishing the reliability of the present method of measuring electrical periods and commending the drop chronograph as an instrument for the precise measurement of small intervals of time.

6. Some conclusions are drawn concerning the serviceability of the method for the absolute measurement of capacities, inductances, or dielectric constants.

7. It is difficult to reconcile the facts of this paper summarized in (1) and (5) with the general practise of assuming the effective inductance of an oscillating coil to be $2/\pi$ times the total inductance. This question waits on more precise information as to the distribution of current and potential in a freely oscillating coil.

COLLEGIATE DEPARTMENT,
CLARK UNIVERSITY,
August, 1916

THE INFRA-RED ABSORPTION SPECTRUM OF NAPHTHALENE AND OF SOME OF ITS MONO-DERIVATIVES IN SOLUTION.

BY A. H. STANG.

INTRODUCTION.

ALTHOUGH much experimental work has already been done on finding the infra-red absorption spectrum of different substances, there remains a very large number of substances whose spectrum is unknown. The results of the infra-red investigations have been very fruitful in solving problems of molecular structure and in testing the energy quantum theory, especially as applied to the theory of specific heats. Thus Bjerrum[1] from a knowledge of the infra-red absorption bands of CO_2 has proposed a solution for the problem of the arrangement of the atoms in the CO_2 molecule, while Nernst and Lindemann[2] have deduced formulas for the variation of the specific heat with temperature that agree closely with experimental data and are based on the location of infra-red absorption bands of the substances in question.

Still more recently Baly,[3] of the University of Liverpool, has proposed a theory which unites the absorption of an organic substance in the infra-red to that in the visible and ultra-violet regions of the spectrum and to the fluorescence and phosphorescence which a body may emit. He has, in particular, predicted from theoretical considerations at what wave-lengths the infra-red absorption maxima of several substances should occur. Among these substances was naphthalene, $C_{10}H_8$.

The object of this paper is then to investigate the absorption of naphthalene as well as that of several of its mono-derivatives and especially to obtain the positions of maxima absorption. Since these substances are solids at ordinary temperatures, it was determined to obtain their absorption curves when in solutions of carbon disulphide and carbon tetrachloride. These solvents were chosen because they are relatively transparent in the infra-red. From these absorption curves the effect of the solvent and of isomerism will be noted. The agreement of these

[1] Bjerrum, Verh. der deut. Phys. Gesell., 16, p. 737, 1914.

[2] Zeit. fur Electro-chemie, 17, p. 867, 1911.

[3] Baly, Astro-phys. Journal, 42, p. 4, 1915. Phil. Mag., Vol. 27, p. 632, 1914; vol. 29, p. 223, 1915; vol. 30, p. 510, 1915; vol. 31, p. 417.

absorption maxima with Baly's theory of absorption will also be tested and some conclusions as to the validity of that theory drawn.

APPARATUS.

A fixed arm spectrometer, provided with the Wadsworth[1] constant minimum deviation attachment was used for the work. Fig. 1 gives a diagram of the apparatus. *A* is an adjustable slit. *B* and *E* are 9 cm. diam. 50 cm. focal length spherical mirrors, silvered on the outside. *C* is a rock salt prism, 8 cm. along each face and 8 cm. high, with a refracting angle of 59° 58′.75, obtained from Brashear. The prism was protected from moisture by having a small dish of anhydrous calcium chloride placed upon it and several others on the spectrometer, which was enclosed in a blackened case. When not in use, the prism was covered with a bristol board box painted inside with paraffine which fitted closely to the spectrometer table. The small weight of this box made it preferable to a heavy glass bell-jar. *D* is the Wadsworth plane mirror, while *F* is a plane 10×10 cm. mirror used to change the direction of the convergent light from *E* so that the filar microscope *J* could be used for reading the 35 cm. diam. circle *K*. *C* is a Rubens thermopile, made up of iron constantin junctions with a fixed slit width of 0.03 cm. A Leeds and Northrup sensitive galvanometer was used on closed circuit in series with the thermopile. The source of radiation *L* was a 220-volt Nernst glower. A spherical mirror *M* focused the light on the slit.

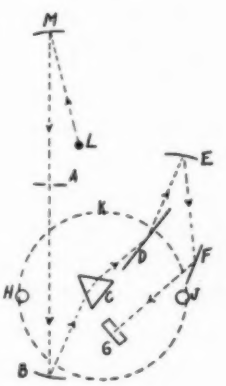

Fig. 1.

The cells used to hold the solutions were made of two plates of polished rock salt about 2 × 4 × .4 cm., between which a clean copper wire bent in U shape was placed. After the plates had been finally polished on an optical grinder with rouge and absolute alcohol, the wire was placed between them and cemented to the plates with SiO_2. The solvents used did not attack any of the materials in the cell. After filling the cell, a glass plate was cemented over the top to prevent evaporation.

A thin wooden block that ran in vertical ways between stops was provided with two openings somewhat smaller than the cells. The cell containing the solution to be investigated was placed over the upper opening. Over the lower opening a duplicate cell, having the same absorption when empty, was placed filled with the pure solvent. The

[1] Wadsworth, Phil. Mag. (5), 38, p. 346, 1894.

elevator was so placed that the cells could be brought in succession exactly to the same position in front of and close to the slit. Between the elevator and mirror M interposed a metal screen which could be operated by the observer.

The naphthalene used was recrystallized from alcohol. The beta naphthylamine was obtained from Merck and Co. The alpha and beta naphthol, the nitronaphthalene and the alpha naphthylamine were very kindly furnished by Professor M. Gomberg of the chemistry department. The solvents were obtained from stock in the laboratory. They were listed as c.p. but since the absorption of the solvents does not enter into the final result, absolute purity is not necessary.

PROCEDURE.

A calibration curve for the rock salt prism was made from values given by Paschen.[1] Since the observation room was always kept at 25° C. the values of the refractive index were corrected for temperature by his correction formula.

Before each set of observations, the instrument was calibrated with an optical setting on the .546 μ Hg line and then tested with the Bunsen flame emission maximum which has been accurately located[2] at 4.40 μ. From the calibration curve, this maximum was always found at a wavelength smaller than 4.4 μ by about 0.4 minute of arc. This discrepancy has been noticed by several investigators[3] who attribute it variously to curvature of the image of the slit and to a real difference between a bolometric and an optical setting of the standard line on the slit. Correction was made by assuming that the Bunsen maximum was at 4.400 μ and displacing the calibration curve by the required amount. It may be noted that the .03 cm. width of slit for a 50 cm. focal length mirror amounts to 2.03 minutes of arc.

After the cells had been placed on the elevator and the zero galvanometer reading taken, the metal screen was raised so that light could pass through the cell which contained the solution. When the maximum deflection had been noted, the elevator was raised until the cell containing the solvent came in front of the slit. The deflection now increased and finally came to rest at a value larger than the first. With a linear thermopile such as was used the percentage of energy transmitted was computed by finding the ratio of the deflection through the solution to that through the solvent.

In general, readings were taken one minute apart on the spectrometer

[1] Paschen, Ann. der Physik, 26, p. 120.

[2] Paschen, Wied. Ann., 52, p. 209, 1894; 53, p. 337, 1894; 60, p. 714, 1897.

[3] Coblenz, Carnegie Inst. Publ., No. 35, p. 20. Paschen, Wied. Ann., 52, p. 209, 1894.

circle but in the neighborhood of an absorption band they were taken closer together so as to accurately locate the minimum of transmission. Table I. is a sample taken at random from observations through one absorption band. It shows the variation of the intensity of the light caused by the absorption of the solute and of the solvent and by the Nernst glower.

TABLE I.

Infra-red Absorption Data. α Nitronaphthalene in CS_2.

1.	2.	3.	4.	5.
Angle from $\lambda = .546\mu$.	Wave-length.	Solution Deflection, Cm.	Solvent Deflection, Cm.	Per Cent. Transmitted.
37.00	7.070μ	1.49	2.01	74.0
38.00	7.160	1.79	3.19	56.1
39.00	7.250	1.42	4.12	33.4
40.00	7.334	1.04	4.58	22.7
40.32	7.360	.93	4.83	19.2
40.62	7.384	.87	4.97	17.5
41.00	7.415	.79	4.93	16.0
41.33	7.440	.84	5.03	16.7
41.61	7.463	.89	5.04	17.7
42.00	7.493	1.00	5.00	20.0
43.00	7.571	1.76	4.95	35.6

The minimum of transmission was always determined from a plot made on a large scale of several values near the minimum. The point where the two lines (Fig. 2) meet was taken as the position of maximum

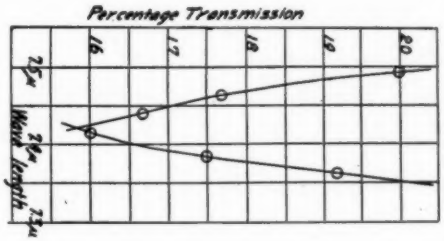

Fig. 2.

α Nitronaphthalene Absorption Band in CS_2 at 7.42 μ.

absorption. No correction has been made for finite slit width and impurity of the spectrum. Several bands were plotted to an enlarged scale and the correction method proposed by Runge[1] applied. The corrected absorption curve was indeed deeper but there was no shift of the position of the maximum comparable to the experimental error.

In the tables giving the positions of maximum absorption are given

[1] Runge, Zeit. für Math. und Phys., 42, p. 205, 1897.

the estimated limits of error in location after account has been taken of errors that may arise in percentage transmission, plot, calibration curve, adjustment test, and change in temperature.

CHEMICAL STRUCTURE OF NAPHTHALENE AND OF ITS DERIVATIVES.

Naphthalene, $C_{10}H_8$[1] consists of two benzene nuclei which have in common two carbon atoms. The accepted formula is given in Fig. 3. The replacement of an H atom in naphthalene gives rise to two isometric mono-derivatives. They are called either α or β derivatives according as the substituent is adjacent to the complex of Fig. 4, common to both groups, or separated from it by a CH group.

α naphthol, $C_{10}H_7(OH) + H_2O$ is represented by the formula of Fig. 5

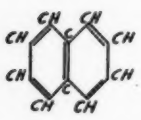

Fig. 3.

Fig. 4.

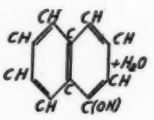

Fig. 5.

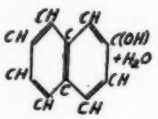

Fig. 6.

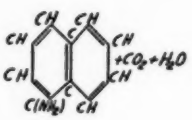

Fig. 7.

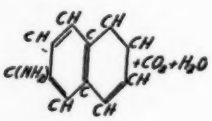

Fig. 8.

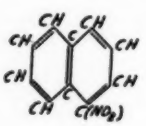

Fig. 9.

Fig. 3. Naphthalene.
Fig. 5. α Naphthol.
Fig. 6. β Naphthol.
Fig. 7. α Naphthylamine.
Fig. 8. β Naphthylamine.
Fig. 9. α Nitronaphthalene.

while β naphthol is shown in Fig. 6. The α and β naphthylamine $C_{10}H_7(NH_2) + CO_2 + H_2O$ correspond to Figs. 7 and 8 respectively and α nitronaphthalene $C_{10}H_7(NO_2)$ has the formula of Fig. 9.

DISCUSSION OF THE RESULTS.

In Fig. 10 are given the transmission curves for naphthalene in CS_2 and in CCl_4 respectively. Table II. gives the location of the absorption maxima. The CS_2 solution gives more bands but except for the band at 3.30 μ the location of the common bands agrees within experimental error. Coblenz[2] has investigated the absorption of naphthalene in CCl_4 in this region and found a band at 3.25 μ. The CS_2 band was however located at 3.30 μ in several solutions, the observations having been taken more than two months apart. This shift from 3.25 μ to 3.30 μ must be due to the effect of the solvent itself.

[1] Richter-Smith, Organic Chemistry, vol. 2, p. 390, 1900.
[2] Coblenz, *loc. cit.*

Since naphthalene consists of two benzene nuclei, it will be of interest to note that Coblenz has found absorption maxima of benzene at 2.7 μ, 3.25 μ, 4.4 μ, 4.9 μ, 5.5 μ, 6.2 μ, 6.75 μ, 7.25 μ, 7.8 μ, 8.67 μ, and 9.78 μ.

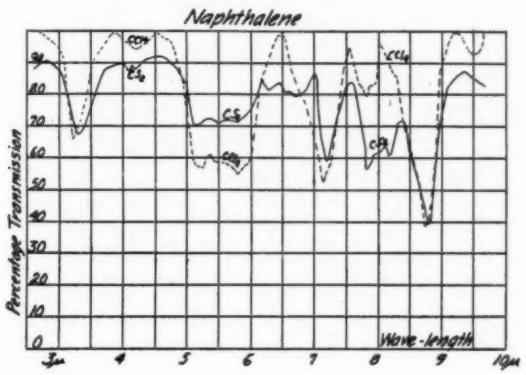

Fig. 10.

TABLE II.

Naphthalene Absorption Maxima.

No.	In CS_2.	In CCl_4.	Wave No.
1	3.30μ ± .03μ	3.25μ ± .03μ	308
2	4.2 .10	4.22 .04	237
3	5.15 .10		194
4		5.27 .03	190
5	5.5 .10		182
6	5.8 .10	5.80 .02	172
7	6.31 .02		159
8	6.75 .10		148
9	7.20 .03	7.18 .03	139
10	7.84 .02	7.84 .03	127
11	8.18 .02		122
12	8.79 .02	8.78 .02	114
13	9.81 .03		102

It is to be noticed that many naphthalene bands occur at the same wave-lengths. These must be due to the vibration of the benzene ring, still present in the naphthalene and are not characteristic of the naphthalene itself.

Baly[1] in his theory has arrived at the conclusion that the wave numbers of the absorption band in the visible or ultra-violet region of the spectrum should be equal to the product of the wave numbers of several infra-red bands by whole numbers. Such a test would serve as a criterion of the worth of his theory. For example, Russel and Lapraik[2] found three

[1] Baly, *loc. cit.*

[2] Chem. Soc. of Lon. Trans., 39, p. 171, 1881.

naphthalene absorption bands, the maximum absorption occurring from .714 μ to .707 μ, from .636 μ to .634 μ, and from .619 μ to .611 μ while the mean wave numbers for these bands are 1407, 1571, and 1626 respectively. The wave numbers of the infra-red bands giving closest agree-

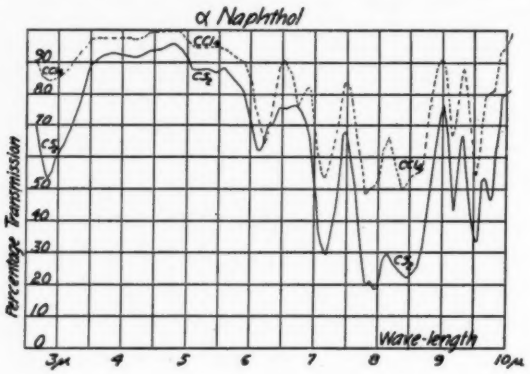

Fig. 11.

TABLE III.

Infra-red Absorption Maxima. Alpha Naphthol.

1.	2. 3.	4. 5.	6.	7.
No.	In CS_2.	In CCl_4.	Wave No.	
1	2.85μ±.03μ	2.87μ±.04μ	286	
2	6.15 .02		162	D
3		6.24 .03	160	
4		6.76 .03	148	ND
5	7.19 .03	7.19 .03	139	ND
6	7.84 .03	7.82 .03	127	ND
7	7.97 .03		125	
8	8.44 .04	8.40 .03	119	D
9	9.17 .02	9.17 .02	109	
10	9.54 .03	9.54 .02	105	
11	9.73 .03		103	D

Purvis at 3068 (2, 5, 6, 9, 11); at 3324 (5, 6, 8).

ment are for 1407, 139 × 10 = 1390, 127 × 11 = 1397, 238 × 6 = 1428; for 1571 there are no close values; for 1626, 102 × 16 = 1632, and 148 × 11 = 1628.

The wave numbers of the other bands found by various investigators in the visible region are given below. The numbers in parenthesis after each wave number refer to the infra-red bands listed in Table II. which are very nearly integral factors of this visible wave number.

By Hartley[1] at 3504 (4, 7); at 3670 (4, 7, 11, 13); at 3801 (3, 6, 7, 11); and at 3922 (2, 11).

[1] Chem. Soc. of Lon. Trans., 47, p. 685, 1885.

By Baly and Tuck[1] at 3125 (4, 8); at 3222 (9); at 3700 (1, 4, 8, 10).

By Guthrie[2] at 3102 (4, 5, 6, 8); at 3167 (7, 11, 13); at 3207 (9, 12); at 3306 (2, 3, 4, 10, 11, 12); at 3488 (4, 7, 9); at 3574 (10, 13); at 3625 (5, 6, 9); and at 3745 (9, 12).

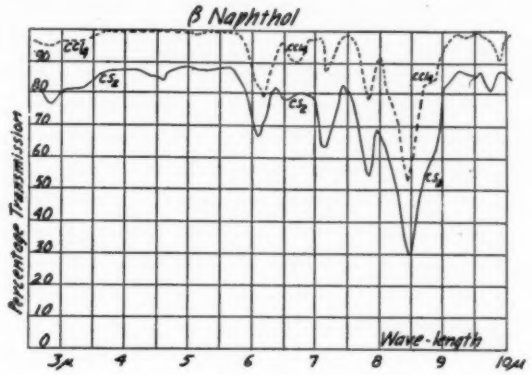

Fig. 12.

TABLE IV.

Infra-red Absorption Maxima. Beta Naphthol.

1.	2.	3.	4.	5.	6.	7.
No.	In CS_2.		In CCl_4.		Wave No.	
1			2.77μ±.10μ		361	
2	2.87μ±.04μ				350	
3	6.12	.03	6.18	.03	163	D
4	6.5	.10			154	
5			6.75	.03	148	ND
6	7.18	.03	7.21	.02	139	ND
7	7.84	.02	7.85	.03	128	ND
8	8.47	.02	8.47	.02	118	D
9	9.72	.02			103	D
10			9.90	.06	101	

Purvis at 2996 (8) at 3712 (4, 5, 7, 9).

[1] Purvis, Chem. Soc. of Lon. Trans., 101, p. 1315, 1912.

From these results it is seen that nearly all of the absorption maxima in the visible and ultra-violet have wave numbers which are integral multiples of several infra-red absorption wave numbers.

The constant frequency difference between bands in the visible or in the infra-red which one might expect from Baly's theory have not been found. This detailed discussion of the results has been undertaken to see if the theory does account for the position of the visible absorption

[1] Chem. Soc. of Lon. Trans., 93, p. 1902, 1908.

[2] Baly, Astrophys. Jour., 42, p. 44.

bands. The proof may be sufficient. It would however be going too far to say that the results are as conclusive as might be desired. From his theory of fluorescence, Baly has predicted that bands would be found at 3.27 μ, 3.86 μ, 4.72 μ, 6.06 μ, and at 8.48 μ. Only the 3.27 μ band has been found, and this band had already been located by Coblenz.

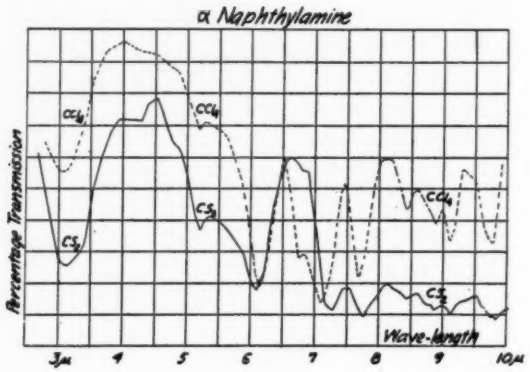

Fig. 13.

TABLE V.

Infra-red Absorption Maxima. Alpha Naphthylamine.

1.	2.	3.	4.	5.	6.	7.
No.	In CS_2.		In CCl_4.		Wave No.	
1	3.09$\mu\pm$.04μ				323	
2			3.18$\mu\pm$.04μ		314	
3	5.19	.03	5.20	.03	194	N
4	6.10	.03	6.18	.02	163	D
5			6.80	.04	147	
6			7.11	.03	141	
7	7.27	.03			138	ND
8	7.76	.03	7.71	.03	129	D
9	8.44	.03	8.47	.02	118	D
10	8.87	.05	8.92	.03	112	
11	9.11	.02	9.12	.03	110	
12	9.80	.04	9.75	.06	103	ND

Purvis at 3174 (9, 11, 12).

The tables given below for the various isomers list the absorption maxima wave-lengths as found with the two solutions and also show the corresponding wave numbers. In column 7, a band is marked N if it is a naphthalene band, and D if the band is found in both the alpha and the beta derivative. At the bottom of each table is given a list of the wave numbers of the known visible absorption bands and following

each wave number, as before in parenthesis, are the numbers of the infra-red bands which are integral factors of the visible wave numbers. The absorption curves for these compounds are given in Figs. 11 to 15.

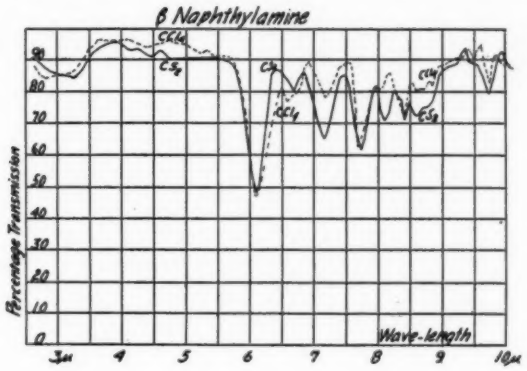

Fig. 14

TABLE VI.

Infra-red Absorption Maxima. Beta Naphthylamine.

1.	2. 3.	4. 5.	6.	7.
No.	In CS_2.	In CCl_4.	Wave No.	
1		2.86μ±.04μ	350	
2	3.25μ±.02μ		308	N
3	4.44 .04	4.44 .04	225	
4	6.13 .03	6.13 .02	163	D
5	6.67 .03	6.64 .04	150	
6	7.18 .03	7.20 .02	139	ND
7	7.73 .02	7.74 .03	129	D
8	8.09 .02		124	
9	8.40 .03	8.41 .02	119	D
10	8.59 .03		116	
11	9.74 .03	9.76 .02	103	ND

Purvis at 2922 (3, 4, 6, 10) and 3470 (5, 6, 9).

CONCLUSIONS.

1. The absorption of naphthalene and of several of its derivatives has been investigated from 2.7 μ to 10 μ in solutions of carbon disulphide and carbon tetrachloride.

2. Bands which are present when one of the solvents is used are nearly always present or indicated by breaks in the curve when the other solvent is used.

3. Although the positions of maximum absorption are generally the

[1] Purvis, Chem. Soc. of Lon. Trans., 101, p. 1315, 1912.

same, in both solutions, the general appearance of the absorption curves may be very different.

4. Naphthalene has several absorption bands which are due to the benzene nucleus.

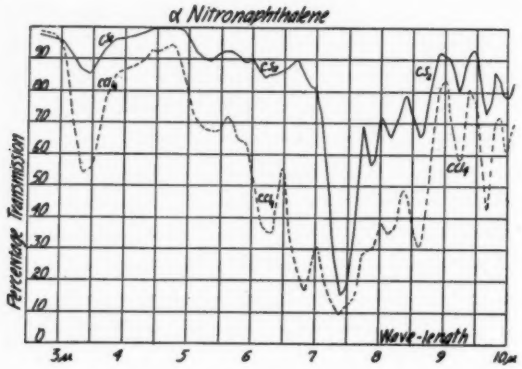

Fig. 15.

TABLE VII.

Infra-red Absorption Maxima. Alpha Nitronaphthalene.

1.	2. 3.	4. 5.	6.	7.
No.	In CS_2.	In CCl_4.	Wave No.	
1	3.44μ±.02μ	3.42μ±.02μ	282	
2	5.29 .04	5.36 .04	188	
3	6.22 .04	6.24 .03	160	
4		6.83 .02	146	N
5	7.42 .02	7.42 .03	135	
6	7.87 .02		127	N
7	8.17 .03	8.20 .04	122	N
8	8.67 .03	8.62 .03	116	
9	9.25 .02	9.23 .03	108	
10	9.67 .02	9.66 .03	103	
11	9.95 .03	9.96 .03	101	

Purvis at 2930 (4, 6, 7, 9, 11).

5. The mono-derivatives of naphthalene have several of the naphthalene absorption bands.

6. There are however many absorption bands present which are characteristic of the derivative itself.

7. Those naphthalene bands occurring in the alpha derivative are generally present in the beta derivative and there are always several other bands common to both derivatives.

8. The infra-red absorption curve of the alpha derivative is however as a whole different from that of the beta derivative in the same solvent.

9. Nearly all wave numbers of absorption bands for these substances in the visible possess, as integral factors, two or more infra-red absorption maxima wave numbers as Baly's theory predicts.

In conclusion, the writer desires to thank Professor H. M. Randall for proposing the subject and also for many valuable suggestions during the progress of the work.

PHYSICAL LABORATORY,
ANN ARBOR, MICH.,
May 2, 1916.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE STANFORD MEETING.

THE eighty-eighth meeting of the American Physical Society was held in Room 370, Physics Department, Stanford University, on Saturday, April 7, 1917, in connection with the meeting of the Pacific Division of the American Association for the Advancement of Science. Morning and after noon sessions were held. Professor Sanford presided.

After brief addresses by Professor J. C. Merriam, chairman of the Pacific Coast Sub-Committee on Research, and Director Campbell, urging cooperation between the various affiliated societies, the Committee on Research, the State Council of Defence, and similar national organizations, it was unanimously voted to pledge such coöperation on behalf of the physicists of the Pacific Coast. Professor Sanford and the Local Secretary were authorized to organize this coöperation.

The following papers were presented:

Physical Significance of Entropy. W. P. ROOP.

Experimental Data Illustrating Fundamental Theorems of Kinetic Theory. W. P. ROOP.

A Note on the Relationship Between Ionizing Potentials and Atomic Charges FERNANDO SANFORD.

An Optical Trough for High School and College Demonstration. R. S. MINOR.

The Relation Between E.M.F. of a two-Solution Cell with Calomel Electrodes and the Difference in Specific Inductive Capacity of the two Electrolytes. DAYTON L. ULREY.

The Effect of Occluded Gases on Photoelectric Emission. L. A. WELO.

The Relation Between Current Strength, Voltage, and Luminosity in the Mercury Arc. H. O. RUSSELL.

The Mercury Arc Pump; The Dependence of its Rate of Exhaustion on Current. L. T. JONES and H. O. RUSSELL.

Note on the Use of the Electrometer for Measuring the Conductivity of Gases. J. A. GILBREATH.

A Study of the Stark Effect. J. A. ANDERSON.

Leading Features of the Electric Furnace Spectra of Calcium, Strontium, Barium, and Magnesium. A. S. KING.

Elimination of Pole Effect from Secondary Standards of Wave-Length. H. D. Babcock and C. E. St. John.

Oscillating Systems Damped by Resistance Proportional to the Square of the Velocity. J. P. Van Zandt.

The Surface Tension of Molten Metals. R. H. Coon and E. E. Hall.

Description of a New Apparatus for Exciting X-Rays. Frank Rieber.

A Mechanical Analogy to Electrostatically Coupled Electric Circuits. W. J. Raymond. (By title.)

Optics of the Microscope. C. W. Woodworth.

Note on Electrical Conductivity. F. J. Rogers.

Rigidity of the Earth from Earthquake Waves. F. J. Rogers.

A Method of Determining J Based on the Adiabatic Expansion of a Gas. J. C. Shedd.

A Hydrodynamical Paradox. Grandison Gardner.

Members were guests of Stanford University for luncheon, and in the evening there was a subscription dinner at Stanford Inn. About thirty-five were in attendance at the sessions and twenty-five at the dinner. At the conclusion of the dinner there was an informal discussion of radiation and atomic theories. At this evening session the following resolution was adopted:

Resolved, that in view of the limited supply of platinum and the great importance of this metal in the manufacture of sulphuric acid, the conversion of ammonia into nitric acid, and many other industrial and scientific purposes closely connected with the national defense, we, the Pacific Coast members of the American Physical Society, urge the State Council of Defense to take immediate steps (1) To conserve our platinum supply; (2) to prevent its waste through the manufacture and sale of platinum jewelry and through other similar unproductive uses; (3) to secure an immediate inventory of our platinum resources; (4) to bring this matter, through suitable channels, to the attention of the National Council of Defence or other branch of the national government, to the end of securing nation-wide conservation of platinum.

The Society also passed a vote of thanks to the authorities of Stanford University for the courtesies and facilities afforded by them.

E. P. Lewis,
Local Secretary for the Pacific Coast.

On the Origin of the Earth's Electric Charge.[1]

By W. F. G. Swann.

MEASUREMENTS of the variation of the penetrating radiation, with altitude, point to the upper atmosphere as the origin of a part of this radiation. The whole of the penetrating radiation is probably of the γ-ray type, but the part which reaches the earth's surface from the outer atmosphere

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

is naturally the most penetrating part. Indeed, it is so penetrating that it passes through a thickness of air which would be equivalent, in absorptive action, to a column of mercury 76 cm. high, if absorption coefficients were simply proportional to density and were independent of material. The γ-ray radiation from the outer layers of the atmosphere will consequently be very "hard," and, in accordance with the known results of laboratory experiments, we must conclude that the negative corpuscles which it emits from the air molecules are emitted almost entirely in the direction of the radiation, and further, that they can have a range in air at least equal to that of the swiftest β-rays from radium products, a range, for example, of 8 meters. The emission of corpuscles by these γ-rays will consequently result, at each point of the atmosphere, in a downward current of negative electricity, which we shall call the corpuscular current. This corpuscular current will charge the earth until the return conduction-current balances the corpuscular current at each point of the atmosphere.

Taking, for the purposes of this abstract, a simplified case where the penetrating radiation considered is all directed vertically downwards, if q is the number of corpuscles liberated per c.c. per second by the penetrating radiation, and h the average distance which a corpuscle travels from its point of origin, the corpuscular current density will be

$$i = qeh,$$

where e is the electronic charge.

If q be taken as 2, which is probably about equal to the number of pairs of ions produced per c.c. per second in a closed vessel as a result of the part of the penetrating radiation in question, and if h be taken as 3 meters, we have

$$i = 2 \times 4.8 \times 10^{-10} \times 800 = \text{about } 8 \times 10^{-7} \text{ E.S.U./cm}^2.,$$

which is just of the order of magnitude of the air-earthcurrent-density, so that on this view, the penetrating radiation from the outer layers of the atmosphere provides a sufficient basis for the explanation of the maintenance of the earth's charge.

The corpuscular current-density, and consequently the conduction current-density, will not necessarily be independent of the altitude, for the factors upon which i depends, viz., the intensity and quality of the penetrating radiation, the number of molecules per c.c. available for possible ionization by the radiation, and the range of the corpuscles set free all alter with the altitude.

A few minor difficulties present themselves if the above view be adopted. Thus, for example, near the surface of the earth, a considerable portion of the whole penetrating radiation comes from the soil, and is directed upwards, but this difficulty disappears when it is remembered that the average "hardness" of the radiation from the soil is very much less than that of the radiation which reaches the earth from the outer layers of the atmosphere. Again, it might appear that the corpuscles set free by the penetrating radiation should, on account of their great energy, produce in the atmosphere many more ions per

second than are actually found to be produced. This difficulty, and others of allied nature become greatly reduced in magnitude, however, when considered in the light of our present knowledge of the action of very swift β-rays when passing through a gas.

The existence of any contribution to the penetrating radiation other than that arising from the radioactive materials in the air and soil has been questioned by some authorities, and it has been supposed that the walls of the ionization chamber are ultimately responsible for such residual ionization as remains when the radiations from the earth and the air are shielded off. To the extent that the " wall effect " arises from an α-particle radiation it could be detected and allowed for, and a probable estimate of the β- and γ-rays radiation accompanying it could be made, if the electroscope were so sensitive that the contributions to the apparent natural ionization were observable as distinct kicks in the recording fiber. The author has succeeded in obtaining the necessary degree of sensitivity for this purpose, and for illustration of the process has obtained photographs showing the rate of emission of the α-particles from a small amount of radioactive material contained in an ionization chamber.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C.,
February 3, 1917.

THE EFFECT OF X-RAYS ON THE LENGTH OF LIFE OF TRIBOLIUM CONFUSUM.[1]

BY WHEELER P. DAVEY.

THE proper use of a physical agent in biological work necessitates the physical measurement of the agent employed. A great deal of work is reported in the literature on the effect of X-rays on various forms of animal life. A study of this literature shows that with very few exceptions the measurements of the dosage have been so incomplete as to render the work of little scientific value. It therefore seemed that the effect of X-rays upon living organisms could be quantitatively studied to some good. The organism studied was the " flour weevil," *Tribolium confusum*. These little beetles are especially adapted to such work as is reported here because they are small, harmless, easy to handle and count in large numbers; they propagate readily, cannot crawl out of glass beakers or small porcelain crucibles, and show little tendency to fly.

It was shown that the effects noticed were really due to X-rays and not to some accidental circumstance, such as static field, ionized air, effect of X-rays on food, etc.

A method has been developed which eliminates the error due to idiosyncrasy, thus making it possible to obtain bio-physical laws of a considerable degree of precision. This method consists in plotting probability curves from the data.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 17, 1917.

If the number of individuals used in the experiments is sufficiently large and if they have been gathered from the same brooder at the same time, then corresponding points on the probability curves may be quantitatively compared. The slope of the probability curves is a measure of the idiosyncrasy of the organism to the physical agent employed.

It was shown that X-rays are able to cause premature death if the dose is larger than a certain threshold dose, but that there is a latent interval between the time the X-rays are administered and the time death occurs. This latent interval depends accurately, for any given individual, upon the logarithm of the X-ray dose, according to the formula

$$Y = A - B \log X,$$

where Y is the length of the latent interval and X is the X-ray dose measured in terms of the electrical input of the X-ray tube, material used as anti-cathode, etc. Voltage, current and distance were kept constant, dosage being varied by varying the time. Too much emphasis can not be given to the necessity of keeping the voltage constant, as the X-ray output of the tube varies greatly with the voltage.

With a standard tungsten target Coolidge tube, operating on sine-wave voltage without X-ray filters, the threshold dose for *Tribolium confusum* is 500 milliampere-minutes at 25 cm. distance at 50 kilovolts, *i. e.*, 500 MAM/25^2 at 50 KV. RMS.

The formula for the length of the latent interval may be easily derived from an extension of the phycho-physic law if the *resistance* rather than the susceptibility of the organism to X-rays is considered.

It is hoped later to present results on the effect of wave-length of X-rays, and on the sterilizing effect of doses below the threshold lethal dose.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.,
SCHENECTADY, N. Y.

THE PASSAGE OF PHOTO-ELECTRONS THROUGH METALS.[1]

BY K. T. COMPTON AND L. W. ROSS.

EXPERIMENTS by Ladenburg and by Rubens and Ladenburg have shown that the electrons liberated from metals by ultra-violet light escape from a surface layer of the metal, of small but measurable depth, and that the electrons are therefore able to pass a certain distance through the metal and still retain sufficient kinetic energy to enable them to escape from its surface.

We have designed a sputtering chamber in which twelve quartz plates may be successively exposed to the cathode deposit and with which we have been able to secure metal films whose thickness is proportional to the time of sputtering.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

These films were then mounted on a rotating disk in a vacuum chamber so that they could be interposed successively in the path of a beam of monochromatic ultra-violet light and the photo-electric current then measured from their emergent side.

After passing through the film, the light fell on a second electrode, producing a current from the latter which was proportional to the amount of light transmitted by the interposed film.

The photo-electric current from the film is given by

$$N = KI_0 e^{-\alpha d} \int_0^d e^{\alpha x} F(x) dx,$$

where I_0 is the intensity of the light entering the film, α is the coefficient of light absorption in the film, d is the thickness of the film and $F(x)$ is the probability that an electron liberated at a depth x in the film may escape from the surface. The form of $F(x)$ has been determined for several simple assumptions regarding the nature of electronic absorption. The simplest assumption is that

$$F(x) = e^{-\beta x},$$

where $1/\beta$ is the average distance which an electron can travel through the metal, normal to its surface, without losing its ability to escape.

Our measurements with platinum films indicate that β is very near the value $0.60(10)^7$ cm^{-1}, which corresponds to a mean free path of $1.67(10)^{-7}$ cm. This is of the order of magnitude of the distance between the atoms in the film. We have detected no difference in the values of β as determined for various wave-lengths, which probably indicates that the escaping electrons lose energy as the result of discrete catastrophies (collisions) rather than by a gradual process.

This value of β is larger than that suggested by the work of Rubens and Ladenburg, but it agrees with the order of magnitude predicted from measurements of the Hall effect in platinum.

If our value of β is correct, it probably disposes of the possibility of explaining the thermionic effect as an integrated photo-electric effect.

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

THE ELECTROMAGNETIC MASS OF THE PARSON MAGNETON.[1]

BY DAVID L. WEBSTER.

IN the Parson magneton theory[2] of the atom, and other non-spherical models, the lack of symmetry of the electromagnetic field makes it evident that the electromagnetic mass of a single magneton or whole atom is not the same for

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] A. L. Parson, "A Magneton Theory of the Structure of the Atom," Smithsonian Miscellaneous Collections, November, 1915.

accelerations in all directions, even when the magneton or atom is at rest. If this were the whole mass of the system, then as Ehrenfest[3] and others have proved, the system when in motion would orient itself in the position of maximum kinetic energy, and thereby give an indication of its absolute motion to any observer moving with it. The principle of relativity therefore demands the existence of another kind of mass that will vary in such a way as to keep the total mass constant. The origin of this mass must be investigated before any definite statements can be made about the relation between the total observed mass and other properties of the magneton.

To take a simpler problem first, if two equally charged spheres are held at a fixed distance a by a string, the mutual electromagnetic mass of the pair is $2e^2/c^2a$ for accelerations along the line of centers, but only e^2/c^2a, or $1/c^2$ times the mutual electrostatic energy, for accelerations across this line. In very rapid motion the tendency of the electromagnetic forces to turn their line of centers toward the direction of motion will be balanced by the effects of the tension in the string, whose direction, according to relativity, is no longer that of the string itself. Therefore it is reasonable to expect this tension to supply the forces opposing acceleration from rest that give the variable "internal mass." It will do this provided that its changes are propagated with the velocity f light according to the following law: The force of the string on each sphere always acts as though the direction of the other were that of the point where it would be if it had not been accelerated since it was last heard from through signals with the velocity c, but as though its distance were still equal to a. Any electrostatic system with this type of internal forces will have a total mass equal to twice $1/c^2$ times its electrostatic energy.

The same idea may be extended to a steady current system, with strings having the peculiar property of adjusting their forces to balance all changes of direction of the line between the elements of current they connect, but with no change of force for a change of distance. The same postulate on propagation of internal forces will then give a variable mass, but in this case the internal mass will be negative and will always exactly balance the mass due to magnetic action. This is not, of course, the only possible system of internal forces, but is suggested tentatively in the absence of any more plausible one. If it is incorrect, it would be possible to make assumptions leading to either positive or negative values of the resultant mass. The zero value therefore does not represent any sort of lower limit, and is as plausible as any other.

In any system of static charges and steady currents, the fields due to the charges and to the currents may be proved to have no mutual electromagnetic momentum. For the case of the magneton, then, we may combine the above suggestions and conclude that its mass when at rest is most probably equal to $2/c^2$ times its electrostatic energy alone.

JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

[1] Ann. der Phys., 1907.

Gyroscopic Effects in the Parson Magneton.[1]

By David L. Webster.

FROM the experiments of Barnett[2] on magnetization of iron by rotation, and those of Einstein and de Haas[3] on the converse of this effect, it appears that the ultimate magnetic particle shows gyroscopic effects of the order of magnitude of those of an electron moving in an orbit of sufficient size and speed to have the required magnetic moment. In view of the difficulties of accounting for stability in the orbits required for observed magnetic phenomena except by the Parson magneton hypothesis, it is necessary to investigate the possible gyroscopic properties of the magneton.

That such properties exist in it appears from a brief consideration of the retarded potentials near a magneton in precessional motion, but the easiest way to obtain an estimate of their magnitude is from a calculation of the moment of electromagnetic momentum[4] around the axis of the ring. Since at all points very near the circumference of the ring the Poynting vector varies approximately as the inverse square of the distance, r, from it, practically all the energy will be concentrated in this region. At such points the electric vector is $2\rho A/r$ and the magnetic vector is $2\rho Av/rc$, or v/c times the electric. Hence the magnitude of the Poynting vector is $v/4\pi$ times the square of the electric vector, and the electro-magnetic momentum per unit volume is twice v times $1/c^2$ times the electrostatic energy per unit volume. That is, the total moment of this momentum is that of a particle of velocity v and mass equal to twice $\frac{I}{c^2}$ times the electrostatic energy of the system. This mass is exactly that of the magneton as found in the previous paper. In an earlier paper on heat radiation[5] the magneton has been treated as a ring structure on which the electric charge is free to flow. Continuing this line of argument we should not expect any gyroscopic effects from the internal forces of the ring, since without rotation on its principal axis no unsymmetrical accelerations are introduced by rotation on an axis lying in its plane. The resultant gyroscopic effect is therefore exactly the same as for an electron of the classical type in an orbit equal in size to the ring and in speed to that of the electricity on the ring, as required by the experiments of Barnett and Einstein and de Haas.

Jefferson Physical Laboratory,
Cambridge, Mass.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] Science *30*, p. 413, 1909, Amer. Phys. Soc., November, 1913, et seq.; Phys. Rev. [2], *6*, 239–70, October, 1915.

[3] Deutsch. Phys. Gesell, Verk., 17.8, pp. 152–170, April 30, 1915.

[4] The theorem that the electromagnetic momentum concept can be used for calculation of moments as well as resultant forces can be proved by a method similar to the ordinary resultant proof.

[5] Proc. Amer. Acad., Jan., 1915.

ON MOSELEY'S LAW FOR X-RAY SPECTRA.[1]

BY H. S. UHLER.

THE experimental data were taken from tables compiled by Siegbahn from his own investigations and those of his co-workers. The constants, a and b, in Moseley's law, $\sqrt{\nu_N} = a + b\,N$, were first calculated by the method of least squares and then the percentage deviations of the computed wave-lengths from the experimental ones were evaluated. This was done for the α and β_1 sub-series of the K series and for the α_1, β_1, and γ_1 lines of the L series. In all cases it was shown that Moseley's law does not hold throughout the entire ranges of wave-lengths now known. The deviations at the extreme wave-lengths were always appreciable and amounted to as much as 13.4 per cent. for the L–β_1 line of arsenic.

The equation $\sqrt{\nu_N} = A + B \cdot N + C/(D - N)$ was shown to reproduce the given data well within the limits of experimental error. The parameters A, B, C, and D are constants for any one sub-series but change from one sub-series to another. By extrapolating with power series in N it was shown that the characteristic radiations from hydrogen probably have wave-lengths less than 300 angstroms. Attention was called to the fact that, so far as X-ray wave-lengths alone are concerned, it is not necessary to assign the series of natural numbers to the chemical elements from hydrogen to uranium, but that there exist a high order of infinity of other sequences of numbers which may be taken to represent the atomic numbers.

YALE UNIVERSITY,
January, 1917.

THE ELECTRICAL DISCHARGE FROM POINTS.[1]

BY JOHN ZELENY.

THE results of some experiments indicate that the electric intensity at the surface of a point discharging a current is independent of the material of the point. Measurements were made in air at different pressures with brass, water, glycerine and methyl alcohol.

The dependence of the electric intensity at the surface of a point when a current can just be maintained, upon the radius of curvature of the point and upon the pressure of the surrounding gas has been investigated for water points in air.

When ions of both signs are supplied by some outside source to the gas near a point, they have a much larger effect upon the current when the point is charged positively than when it is charged negatively.

When points show a retardation in the commencement of a current, this current begins to flow at any voltage above a certain minimum value, only a

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

certain interval of time after this voltage has been applied, which interval of time decreases rapidly as the value of this applied voltage increases above the minimum value. On lowering the voltage the current stops suddenly at nearly this last value of the voltage. The minimum current that can be maintained with such sensitive points diminishes regularly with the pressure of the gas.

When single alpha rays enter the region near a charged sensitive point, the range of voltage over which they can produce the large temporary increase in ionization, which is used in counting these particles, includes the minimum potential for which a current can be maintained from the point.

YALE UNIVERSITY.

ON THE PRODUCTION OF SOFT X-RAYS BY SLOW MOVING ELECTRONS.[1]

BY H. M. DADOURIAN.

ELECTRONS from a hot line-cathode were made to impinge against an aluminium target after falling through accelerating electric fields varying from 20 volts to 1,000 volts. The properties of the resulting X-rays were studied by the " photo-electric " effect of the X-rays.

Evidence was obtained of the generation of soft X-rays by electrons impinging against an aluminium target with velocities corresponding to 20 volts.

If a stream of electrons impinge against a solid with velocities corresponding to a voltage V only a very small proportion of the resulting X-rays have the maximum frequency given by the relation

$$Ve = h\nu$$

When soft X-rays fall upon metal plates from 5 to 20 per cent. of the rays are scattered or reflected.

Thin films of celoidine have been obtained which transmitted from 20 to 40 per cent. of the X-rays.

The " photo-electric " current was found to be proportional to the cathode ray current.

SLOANE LABORATORY,
YALE UNIVERSITY.

EXPERIMENTS CONCERNING " MAGNET-PHOTOGRAPHY."[1]

BY L. A. BAUER AND W. F. G. SWANN.

IN the number of the Scientific American for November 4, 1916, experiments by F. F. Mace are recorded showing that certain articles possess the power of affecting a photographic plate when they are allowed to remain in contact therewith under the influence of a magnetic field. A confirmation of some of Mace's results was recorded by the authors at the meeting of the American

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

Physical Society in December, 1916, and the present paper deals with the continuation of the investigation.

A large number of experiments have been performed at the laboratory of the Department of Terrestrial Magnetism, some with permanent and some with electro-magnets. The details of these experiments are too numerous to be recorded in abstract, but it is concluded that while a magnetic field is not essential to the production of photographs of the kind cited, it probably has an influence in intensifying the effect.

As W. J. Russel and others have shown, wood possesses the power of affecting a plate when allowed to remain in contact with it for some time. The action of the wood is primarily due to the resin in it, and is considerably enhanced if the latter, before use, is stimulated by light.

The action of the resin extends beyond the place of its immediate contact, and in the present experiments, when some stimulated resin was placed on a plate in company with a metallic article, it was found that the image of the article appeared without the employment of a magnetic field, and simply because of the shielding action of the metal on the plate. On evacuating the vessel which contained the article, the effect of the resin extended more uniformly over the plate and was less concentrated in the immediate vicinity of the resin than before; as a result, the image of the metallic article was much more uniform in the case of a vacuum than in that of air.

It has been shown by Russel that metals, especially zinc, when freshly sandpapered, possess the power of influencing a photographic plate when in contact therewith; but in the experiments here described, this action is overpowered by the shielding action of the metal as regards the influence of the resin.

When, in the experiment with resin and the metallic article described above, the resin was replaced by pieces of wood, no apparent image of a set of *metallic articles* appeared after a 7-day exposure in a vacuum, but the absence of effect in this case is probably only one of degree, since the wood contains resin. It is interesting to observe, however, that when the same experiment was carried out under the influence of a magnetic field, distinct impressions were obtained in a vacuum after an exposure of 6 days.

The experiments are being continued.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C.,
February 3, 1917.

THE CRYSTAL STRUCTURE OF ALUMINIUM AND SILICON.[1]

BY ALBERT W. HULL.

THE author's method of applying X-ray crystal analysis to microscopic crystals has been briefly described in this journal in connection with the crystal structure of iron.[2] It consists in sending a narrow beam of mono-

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] PHYS. REV., 9, 84, January, 1917.

chromatic X-rays through finely powdered crystals, and photographing the diffraction images of the slit, produced by reflection in all the possible planes belonging to the crystal structure. To make sure that all orientations are equally represented in the aggregate of tiny crystals, the powder is made as fine as possible and kept in continuous motion during the exposure. The positions and intensities of the lines on the photograph are compared with the theoretical values for different possible arrangements of atoms, until an arrangement is found which represents completely the observed lines.

In the preliminary calculations it is assumed that the scattering electrons are concentrated at the centers of the atoms. This gives wrong intensities for the lines, but causes no error in their positions. The relative intensities of the line should be completely accounted for by the position and condition of the electrons in the atoms, and since the number of lines that can be photographed is very large, this offers a powerful means of testing theories of atomic structure. It is hoped to report the results of such an investigation in the near future.

Aluminium.

Fine filings from chemically pure sheet aluminium, in a thin-walled glass tube of 2 mm. diameter, were exposed for 3 hours to a narrow beam of rays from a Coolidge tube with molybdenum target, running at 40,000 volts, 9 milliamperes. The diffracted rays were photographed on Eastman X-ray film with calcium tungstate intensifying screen, bent in a circle of 11.1 cm. radius. The rays were filtered, before striking the crystals, by a paste of powdered zircon crystals 0.3 mm. thick, which completely absorbs the β lines of the K radiation and leaves, as a practically monochromatic radiation, the α doublet, $\lambda = .710$ Å. and $\lambda = .714$ Å. The reflection of this doublet in the different crystal planes gives the lines which appeared on the photograph.

They are tabulated in the accompanying table, together with the corresponding spacing (calculated from the angle of the line and the wave-length) and the theoretical spacing for the assumed lattice. The last column gives the indices of the plane which produced each line.

Aluminium has usually been considered to belong to the isometric system. The X-ray data shows that it must be tetragonal. The unit of structure is a *centered rectangular prism*, with square base 2.85 Å. × 2.85 Å. and height 4.05 Å., *with an atom at each corner and one in the center of the prism.* The structure is thus similar to that of iron, except that the unit is a rectangular parallelopiped instead of a cube. The number of atoms per unit parallelopiped is

$$n = \frac{[\text{density}] \times [2d_{100}]^2 \times [2d_{001}]}{\text{mass of atom}} = \frac{2.70 \times \overline{2.85}^2 \times 4.05}{2.69 \times 1.663} = 1.99.$$

This is equal, within the limit of error of the measurements, to 2, which is correct for the assumed structure.

Silicon.

Fine crystalline silicon, pulverized in a steel mortar, was mounted in the same manner as aluminium, and exposed for 4 hours to filtered Mo rays at 32,000

volts, 8 milliamperes. The results are given in the table. The lattice is identical with that of diamond, *i. e.*, two-face centered lattices intermeshed, the one being displaced with reference to the other along a cube diagonal a distance of $\frac{1}{4}$ the length of the diagonal. The side of the cube is 5.43 Å., as compared with 3.56 Å. for diamond, and the distance between nearest adjacent atoms is 2.35 Å. as compared with 1.54 Å. for diamond.

The number of atoms per unit cube of the lattice is

$$n = \frac{\rho[4d_{100}]^3}{M} = \frac{2.34 \times \overline{5.43}^3}{28.1 \times 1.663} = 8.00,$$

which is correct for the assumed lattice.

Aluminium.						Silicon.					
Distance of Line from Center, Cm.	Spacing of Plane, Ångströms.		Intensity of Line.		Indices of Plane.	Distance of Line from Center, Cm.	Spacing of Plane, Ångströms.		Intensity of Line.		Indices of Plane.
	Experimental.	Theoretical.	Experimental (Estimated).	Theoretical.			Experimental.	Theoretical.	Experimental (Estimated).	Theoretical.	
3.45	2.33	2.33	100	100	011	2.58	3.13	3.14	100	100	111
3.99	2.02	2.02	60	64	001	4.21	1.93	1.93	80	344	110
					110	4.97	1.64	1.64	75	148	311
5.67	1.426	1.426	50	86	100			1.57	0	0	111 (2)
					112	6.00	1.36	1.356	25	118	100
6.68	1.21	1.21	60	142	211	6.54	1.25	1.25	45	108	331
					013	7.39	1.11	1.11	50	370	211
6.95	1.17	1.17	20	45	011 (2)	7.84	1.05	1.04	40	116	511
8.09	1.01	1.01	5	28	001 (2)						111 (3)
					110 (2)	8.59	.96	.96	20	152	110 (2)
8.86	.93	.93	25	101	301	8.98	.92	.92	30	144	531
					213	9.66	.86	.86	25	262	310
9.11	.90	.90	25	96	111	10.01	.83	.83	10	63	533
					114			.82	0	0	311 (2)
					130	10.62	.79	.79	5	77	111 (4)
10.05	.82	.82	10	85	132	11.00	.76	.76	10	110	711
					102						551
10.66	.78	.78	15	104	011	11.58	.73	.73	20	414	321
					105	11.90	.71	.71	15	148	731
					321						553
11.75	.71	.71	2	34	100 (2)			.66	0	47	100 (2)
					112 (2)			.64	0	45	733
12.30	.68	.68	4	128	411	13.33	.64	.64	5	172	411
					233	13.60	.63	.63	5	98	751
					215						111 (5)

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.

"It will be noted that the structure here given for Aluminium is identical with a face-entered cubic lattice of side 4.05 A°, a fact which had escaped notice, owing to a numerical error in preliminary calculations."

Ionization Potentials in Hydrogen, Nitrogen and Nitrous Oxide.[1]

By Fred M. Bishop.

THE ionization potential of certain gases has been studied under conditions which tend to minimize the photo-electric effect on the receiving electrode due to radiations in the tube.

Two different forms of apparatus were used, one a modification of the one used by Franck and Hertz in which plate electrodes were substituted for cylindrical ones. The other piece of apparatus contained an aluminium plate and quartz window and photo-electrons were used.

Ionization in hydrogen began at 11 volts and a second and more intense type set in at about 15.8 volts. No ionization could be detected in nitrogen until an accelerating potential of 7.5 volts was applied. The method was extended to nitrous oxide to see if the combination of one atom with another in a compound had any effect on its ionizing potential. Curves for nitrous oxide did not differ appreciably from those in nitrogen.

Yale University.

New Methods for Counting the Alpha and the Beta Particles.[1]

By Alois F. Kovarik.

THE principle of the method is the discharge to a point properly sensitized and is due to Geiger. The method of observation used by Geiger and by myself with L. W. McKeehan in former researches has been the optical method: observing the sudden movement of the fiber in a string electrometer when an alpha or a beta particle caused a flow of electricity to the point. The C. T. R. Wilson's inclined leaf electroscope has also been adapted for the purpose.

I have found recently a very convenient instrument for the same purpose to be Zeleny's electroscope.[2] In the latest form, the leaf is hinged and the instrument is very sensitive. Its sensitiveness is increased by using a potential difference between the plate and the leaf and case. The leaf is connected directly to the discharging point. The apparatus so arranged is very convenient also for lecture demonstrations.

Another method of observation is based on the utilization of the current to the point in connection with a sensitive telephone. If the discharging point is connected to earth through a sensitive telephone, the discharges are made just audible. The effect was greatly magnified by adapting an audion amplifier. The discharging point is connected to earth through the primary coil of a small induction coil (telephone amplifier). One end of the secondary coil is connected to the grid of the audion bulb while the other end is connected to one plate of a

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] Phys. Rev., 32, p. 255, 1910.

condenser, the other plate being earthed. The plate of the audion is connected in series with a sensitive telephone and a battery of about 50 volts, the negative end being earthed. The filament is lighted from a 6-volt battery whose positive pole is earthed. When the number of particles counted is fairly large, this method seems even more accurate than the observation of the fiber of the string electrometer.

Experiments are under way to modify this method so that the particles may automatically register themselves on a chronograph.

It is not difficult to make a point which will respond to alpha particles and give their number accurately but for the beta particles, greater care must be taken to have the point carefully made and to have it quite sensitive. The experiments under way show that their number is also correctly given when proper precautions (see former publications) are taken. Gamma rays also may be studied by these methods and just what these observations signify is being investigated.

YALE UNIVERSITY.

ON THE MEASUREMENT OF "h" BY MEANS OF X-RAYS.[1]

BY F. C. BLAKE AND WILLIAM DUANE.

IN April, 1915, Duane and Hunt presented a paper to this Society describing experiments with X-rays in which it was shown that the equation

$$Ve = h\nu$$

(where V represents a constant voltage applied to the tube, e, the elementary charge and h, Planck's action constant) holds true when, and only when ν represents the *maximum* frequency of the X-rays produced.[2]

In fact we may define "h" to be the kinetic energy of an electron divided by the maximum frequency of the radiation it can produce, when it hits an atom.

The law furnishes us with one of the most accurate methods we have of determining h, e being supposed known.

The authors of this paper have begun a series of experiments designed to determine the value of h under widely different conditions, and also the variation in the amount of X-radiation when different substances are used as targets.

Many improvements have been made in the methods of measurement. A new spectrometer has been designed, with Mr. Franklin L. Hunt's assistance, which enables us to read the positions of both the crystal and the ionization chamber to within 10″ of arc, corrected as usual for eccentricity by means of two verniers at the ends of a diameter. As we make measurements with the

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] Dr. D. L. Webster and Dr. A. W. Hull have since verified the law under different conditions.

ionization chamber first on one side and then on the other of the zero line, thus avoiding the necessity of determining the zero point at all, and as the smallest angle measured amounts to about 6°, the error from reading angles does not exceed 1/10 per cent. for each individual determination.

The electrical current through the X-ray tube comes from a high tension storage battery of 20,000 cells. An electrostatic voltmeter (kindly loaned to us by Dr. Webster and Dr. Clark) measured the voltage applied to the tube. This we calibrated by means of a measured current flowing through a manganin resistance of 898,000 ohms. To avoid errors due to changes in the constants of the voltmeter we calibrated it over the range used each time we made a determination, thus referring each measurement of a voltage to that of a current and a resistance. We estimate that each voltage measurement is correct to within 1/10 per cent.

The errors in measuring the voltage and the " glancing angle " are small as compared with the error that depends upon the interpretation one gives to the shape of the curve near the point at which the radiation vanishes. (Slides were shown illustrating these curves.) The shape of the curve is largely due to the widths of the slits and source of the rays (focal spot on X-ray target).

Using Millikan's value of $e = 4.774 \times 10^{-10}$ our values of h vary from 6.507×10^{-27} to 6.559×10^{-27} according to the estimates we make of the effects of the widths of the slits and source.

HARVARD UNIVERSITY,
February, 1917.

DETERMINATIONS OF LUMINOUS EFFICIENCY AND THE MECHANICAL EQUIVALENT OF LIGHT.[1]

BY W. W. COBLENTZ AND W. B. EMERSON.

IN connection with the value of the mechanical equivalent of light as determined by using the visibility curve of the average eye, the radiation constants of a black body, and the brightness of a black body at various temperatures,[2] it was of interest to obtain further data upon this important constant. The present measurements were made upon a standardized vacuum tungsten lamp obtained from the Photometric Section of this Bureau. The lamp was operated (stationary) at 1.23 watts per candle, giving 26.5 c.p. or 26.5×10^{-4} lumens per $\overline{\text{cm}}^2$. The total radiation was 293.5×10^{-6} watt per $\overline{\text{cm}}^2$. at a distance of 1 m. from this lamp.

The radiant luminous efficiency was determined by means of a linear thermopile (of bismuth and silver) and a luminosity screen having a transmission curve which coincides very closely with visibility curve of the average eye.[3]

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] PHYS. REV. (2), 9, p. 87, 1917.

[3] The constituents of the solution are:

Cupric chloride ($CuCl_2.2H_2O$)	5.7 grams
Cobalt ammonium sulphate [$Co(NH_4)_2(SO_4)_2.6H_2O$]	1.2 "
Potassium chromate (K_2CrO_4)	0.16 gram
Nitric acid (HNO_3). 1.3 c.c. sp. gr. 1.05	0.123 gram
Water (H_2O)	100 mols (c.c.)

This solution is used in a glass cell 1 cm. in thickness backed by a cell of water 3 to 4 cm. in thickness.

The various corrections were of course applied. The weighted mean value of the luminous efficiency is $L_r = .0142$ per cent. for a "40-watt" vacuum tungsten lamp operated at 1.23 watts per candle.

The mechanical equivalent of light is obtained from a knowledge of the total radiation, the luminous efficiency, and the candlepower of the lamp used. The luminous equivalent is $(293.5 \times 10^{-6} \times .0142 =) 4.16 \times 10^{-6}$ watt per cm^2. Equating this to the candlepower data (26.5×10^{-4} lumens $= 4.16 \times 10^{-6}$ watt) gives a value of 1 *lumen* = .00157 *watt* of radiant energy of maximum visibility, which is in close agreement (3 per cent.) with the value given in the main part of the paper,[1] which gives further checks upon this value.

Our conclusions are therefore in agreement with Ives and Kingsbury[2] that the value of the mechanical equivalent of light is close to 1 lumen = .0016 watt of radiant energy of maximum visibility.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
January 24, 1917.

THE ELECTRO-MAGNETIC MASS OF THE PARSON MAGNETON.[3]

BY C. DAVISSON.

EXPRESSIONS are obtained for the electro-magnetic mass of the Parson magneton for small velocities parallel and at right angles to its axis. These, in general, are not the same and vary with the velocity of the charge in the ring. It is shown, however, that when the velocity of the charge in the ring is equal to that of light the directional variation of the mass disappears. The masses parallel and at right angles to the axis are the same and given by

$$m = \frac{1}{c^2}\frac{e^2}{8\pi\rho}\overline{K},$$

where c = velocity of light,

e = charge of the magneton (Heaviside units),

ρ = radius of ring,

[1] PHYS. REV. (2), 9, p. 87, 1917.

[2] PHYS. REV. (2), 8, p. 254, 1916.

[3] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

$$\overline{K} = \frac{2}{\pi}\int_0^{\pi/2} \frac{d\theta}{(1 - \bar{u}\sin^2\theta)^{\frac{1}{2}}},$$

$$\bar{u} = 1 - \frac{r^2}{4\rho^2},$$

and

r = radius of cross-section of ring.

CARNEGIE INST. OF TECHNOLOGY,
PITTSBURGH.

INTENSITIES OF X-RAYS OF THE L SERIES AS A FUNCTION OF VOLTAGE.[1]

BY DAVID L. WEBSTER AND HARRY CLARK.

IT was shown by one of us[2] that all of the lines of the K-series of the X-ray spectrum of rhodium appear only at voltages greater than a definite critical value. This is the quantum voltage corresponding to a wave-length equal to or slightly shorter than the shortest line of the series. Above this critical voltage all of the lines of the series were found to increase in the same ratio with increase of voltage. In this paper are presented the results of preliminary experiments of a similar nature on the L-series of platinum. The apparatus and methods of work were similar to those referred to above.

Friman's[3] graphs of the square root of frequency against atomic number may be divided into two classes according to slope and curvature; thus suggesting that the L-series consists of two groups, L_1, containing lines $\alpha_1\ \alpha_2\ \beta_2$ and β_3, and L_2, containing $\eta\ \beta_1\ \beta_3\ \beta_4\ \gamma_1\ \gamma_2\ \gamma_3$ and γ_4.

We have studied the intensities of $\alpha_1\ \beta_2\ \beta_1\ \beta_4$ and γ_1 as functions of voltage and have found that α_1 and β_2 have the same critical voltage. 11.45 k.v. $\pm$ .05. This is the quantum voltage for the line β_3, the shortest line of the (supposed) series. $\beta_2\ \beta_4$ and γ_1 have the same critical voltage, 13.20 k.v. $\pm$ .05, the quantum value for a wave-length between γ_1 and γ_2. Accordingly $\gamma_2\ \gamma_3$ and γ_4 (not yet examined) belong to neither L_1 nor L_2.

We have found also that the intensities of the lines of both series are given by

$$I = k(V - V_c)^{\frac{3}{2}},$$

where V_c is the critical voltage of the series to which the line belongs and k is constant for any one line. The relation between the values of k for various lines has not been determined. This formula holds also for the K-series lines of rhodium according to the experiments referred to above.

The value of h derived from our experiments is 6.53×10^{-27} erg. seconds.

HARVARD UNIVERSITY.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] PHYS. REV., June, 1916.

[3] Phil. Mag., Nov., 1916.

APPLICATION OF THE PHOTO-ELECTRIC CELL AS A PYRHELIOMETER.[1]

BY W. W. COBLENTZ.

ABOUT 10 years ago Ångström[2] described a new method for studying solar radiation, based upon diffusion of the violet and ultra-violet rays by the atmosphere. He covered his pyrheliometer with a blue glass. In this manner he was able to study atmospheric diffusion of solar radiation independently of the water vapor present, and obtained some very interesting data on the variation of the solar constant.

In view of the fact that his instrument was lacking in sensitivity, a potassium photoelectric cell (made by Dr. Kunz) was tested in view of its high sensitivity in the blue-violet. It was found that the galvanometer deflection was not proportional to the stimulus; but for high intensities there were indications that the correction might be found quite uniform. A cell of improved construction gave direct proportionality.

The cell was found too sensitive (battery only 2 volts) to be used with a d'Arsonval galvanometer. Using a higher voltage and milliammeter, the current which passed through the cell, when exposed (through blue glass to reduce the intensity) to the sun was sufficient to damage the cell.

The experiments thus far indicate that by using a less sensitive photo-electric substance the application of this instrument as a pyrheliometer deserves further study.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
February 3, 1916.

ON THE INSTABILITY OF ELECTRIFIED LIQUID SURFACES.[1]

BY JOHN ZELENY.

THE behavior of a liquid whose surface has become unstable owing to electrification has been studied by means of photographs taken with instantaneous illumination, and also from so-called moving pictures taken at the rate of 800 per second.

When the unstable condition is reached on an alcohol drop a fine thread of liquid about 0.004 mm. in diameter is pulled out of the surface at the rate of about 8 meters per second. The thread of liquid breaks up into minute droplets at the rate of about a million or more per second. These droplets spread out in their flight so as to fill a conical volume and the mist thus formed appears colored by transmitted and by reflected light.

The length of the threads obtained from an alcohol surface is usually under a millimeter but threads 15 mm. long have been obtained from a glycerine surface. With a surface of the order of one square millimeter only one thread

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] Ångström, Nova Acta Reg. Soc. Sci. Upsala, IV, Vol. 1, 1907.

is usually formed at any instant, but with a sufficiently high voltage several threads may be produced at once.

When highly electrified drops are made to issue from the end of a capillary tube the electrical field in the neighborhood undergoes rapid changes in direction and magnitude because of the flying material and the liquid masses assume some curious shapes which undergo modifications with extreme rapidity.

YALE UNIVERSITY.

BIO-PHYSICS.[1]

BY WHEELER P. DAVEY.

THE application of chemistry to the study of biological problems has been of far-reaching importance. There is every reason to believe that a similar application of physics will be of like value. An outline of the present knowledge of the physics of the living cell will be given, together with some of the theory of the physical structure of cells. The effects of surface tension, adsorption, diffusion and osmosis upon the life of cells will be given. The relation of other branches of physics to biological problems—especially the study of medicine—will be given.

RESEARCH LABORATORY,
GENERAL ELECTRIC CO.

TUNGSTEN ELECTRODE LOSS IN THE HYDROGEN ARC.[1]

BY G. M. J. MACKAY AND C. V. FERGUSON.

DUFFIELD[2] has made a great many determinations of the loss in weight of the electrodes of the carbon arc under varying conditions, and has concluded that the loss of an atom of carbon from the cathode is accompanied by the simultaneous emission of four electrons. From our experience with arcs between tungsten electrodes in various gases we are led to believe that there is no such amount of tungsten lost relative to the transfer of current, and that the phenomena involved do not necessarily require the emission of anything but electrons from the negative electrode.

A quantitative experiment to illustrate the magnitude of the different factors was performed by weighing the tungsten buttons used as electrodes of an arc in hydrogen at approximately 300 mm. pressure before and after operation.

The apparatus consisted of a spherical glass bulb about 12 cm. in diameter. Tungsten leads were sealed in, tipped with removable tungsten buttons of approximately equal size and about 2 mm. apart. The arc was operated from a 500-volt D.C. machine, and was started by heating the electrodes by an auxiliary high potential transformer. The arc was run continuously for 1,265 minutes, the volt ampere characteristics being

[1] Abstract of a paper presented at the New York meeting of the Physical Society, February 16–17, 1917.

[2] Proc. Roy. Soc., *92*, pp. 122–143, 1915.

Initial.....201 volts, 1.40 amperes,
Final.....245 volts, 1.40 amperes.

The dimensions of the electrodes were

	Positive.	Negative.
Length	3.17 mm.	3.81 mm.
Diameter	3.17 "	2.03 "
Initial weight	0.4435 g.	0.3785 g.
Final weight	0.4085 g.	0.4026 g.
Difference	−0.0350 g.	+0.0241 g.

The cathode had thus gained 75 per cent. of the material lost by the anode. This interchange could not be accounted for by convection currents since the arc being horizontal, the tendency would be for the metal vapor to go upwards to the top of the bulb. Apparently, then, the tungsten particles given off by the anode become positively charged and migrate towards the cathode.

The temperature of the electrodes was measured by an optical pyrometer of the Holborn-Kurlbaum type as follows:

	Positive.	Negative.
Back shoulder	2681° K.	2469° K.
Middle of side	2734	2681
Front shoulder	3056	3200
Face	3151	3112

Assuming the whole of the weight lost to have come from the face of the anode, the loss in grams per square centimeter per second would be $7.9 \cdot 10^{-6}$ which corresponds to the rate of evaporation in vacuum at 3040° K., less than the temperature found. At 3151° K. the rate would be $28 \cdot 10^{-6}$.

It might be expected that the normal rate of evaporation would be reduced by the gas present as in the nitrogen-filled lamp. There is evidence, however, that hydrogen lowers the rate of evaporation to a lesser extent than nitrogen which at atmospheric pressure reduces the rate to about 1 per cent. of the rate in vacuum. It is particularly difficult, also, to entirely eliminate water vapor from hydrogen, so that some loss of tungsten may be attributed to attack from this source.

It would therefore appear that the following conclusions may be drawn:

1. The cathode though at a very high temperature gains more weight than it loses.

2. The anode loses weight at a rate not appreciably greater than would occur from normal evaporation of the metal due to its temperature alone.

3. The metal leaving the anode becomes positively charged and migrates to the cathode. The number of atoms carried for each unit charge is about 10^{-4}.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.

A NOTE ON THE RELATION BETWEEN IONIZING POTENTIALS AND ATOMIC CHARGES.[1]

BY FERNANDO SANFORD.

THE ionizing potentials necessary to produce the single-line spectra of a number of elements have been determined. It was shown by the present writer at the last meeting of the Pacific Physical Society that on the assumption that the electrons which radiate light and X-rays are moving in approximately circular orbits about a positive central charge, the magnitude of this central charge may be computed from the formula $Q = 2.95 \times 10^{-12}/\sqrt{\lambda}$, where λ is the wave-length of the radiation emitted. It is accordingly possible to calculate the central positive charges for the electrons which give the single-line spectra and to compare these charges with the potential necessary to set up this radiation.

In the following table are given the wave-lengths of the single-line spectra

I.	II.	III.	IV.	V.
Element.	λ.	Q.	V Obs.	V Calc.
Mercury.....	2537	5.86	4.89	4.81
Zinc.....	3076	5.32	4.04	4.06
Cadmium.....	3260	5.17	3.81	3.85
Magnesium.....	2852	5.53	4.35	4.35
Calcium.....	4227	4.54	2.94	2.97
Strontium.....	4608	4.35	2.69	2.70
Barium.....	5536	3.96	2.24	2.16

as collected by Langmuir in a paper in Trans. Am. Electro-Chem. Soc., XXXIX., 152, with their central positive charges calculated from the above formula, while in the fourth and fifth columns are given the ionizing potentials in volts as determined experimentally and as calculated from the equation $V = (Q - 2.42)\,1.4$ where V is the ionizing potential in volts and Q is the positive charge in electrostatic units multiplied by 10^{10}.

It will be seen from the above table that when the ionizing potentials are plotted against the positive charges of the respective atoms a straight line relation will be found.

This is even more strictly true if the theoretical ionizing potentials of the single line spectra of the alkali metals as given by Langmuir in the same paper are plotted against their respective central charges.

A STUDY OF THE STARK EFFECT.[1]

BY J. A. ANDERSON.

FROM the standpoint of astrophysics it is highly important to be able to examine for Stark effect the lines of such elements as Fe, Ni, Cr, Ti, V, etc., elements which are specially prominent and important in the solar spec-

[1] Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

trum. Such work requires fairly high dispersion, for the spectra are all rich in lines, and consequently the source must be reasonably bright. An apparatus which works quite well has been constructed recently, and with it some results of interest have already been obtained. The essential feature of the apparatus is the cathode, which is a metal cylinder 12 mm. in diameter and about 15 mm. long, surrounded by a silica tube which extends about 2 cm. beyond the cathode. The luminous discharge is thus confined within the silica tube and is observed through a slot extending from the end of this tube to the surface of the cathode. This cathode and a suitable anode are inclosed in a bell-jar which is exhausted until the dark-space has a length of 5 to 8 mm. Direct current of from 10 to 100 milliamperes at 6,600 volts supplied by a set of high-potential generators is used, the maximum field strength being about 16,000 volts per cm. When the current passes, the cathode heats up rapidly and can easily be melted; in general, however, the current is so regulated that the temperature of the cathode is between 600° and 1000° C. In the region λ 3800 to λ 4600, most of the lines of the metals enumerated above can be recorded in an exposure of 15 minutes, using a dispersion of about 4 Ångström units per mm.

The majority of the lines of these metals show no appreciable Stark effect. Exceptions are:

1. The lines λλ 4098, 4111, 4129, 5275, 5298, and 5329 of chromium, which break up into components in a complicated way, the maximum separation being about one fourth that given by Hδ.

2. A group of about a dozen lines of nickel in the region λ 3940–λ 4027; these do not break up into components, but the wave-length is changed; *i. e.*, each line shifts bodily either to the red or to the violet.

3. Five faint iron lines in the λ 4100 region, which behave like the nickel lines.

An effect similar to that for nickel has been found for 32 lines of the secondary spectrum of hydrogen in the region λ4370–λ 4565.

LEADING FEATURES OF THE ELECTRIC FURNACE SPECTRA OF CALCIUM, STRONTIUM, BARIUM, AND MAGNESIUM.[1]

BY ARTHUR S. KING.

THE furnace spectra of calcium, strontium, barium, and magnesium have been studied by the method already employed for several other elements, the spectra produced at temperatures ranging from 1650° to 2500° being photographed for the interval λ 2600 to λ 7200. Material is thus obtained for a classification of lines according to the temperature at which they appear and the rate of increase in intensity with rise of temperature. Certain lines can be selected which are useful as temperature indicators in the spectra of other sources, lines strong at the lowest furnace temperature being of special interest. The fact that a considerable proportion of the lines in these spectra are known

[1] Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

to be connected by series relations furnishes an opportunity to note the behavior of series lines at various temperatures. While the members of the same series are affected similarly, large differences in behavior are found between separate series. A triplet series of calcium was found which is favorably situated to test the change, among the members of the same series, of relative intensity with wave-length at different temperatures. As the temperature rose, the violet members of the series were clearly strengthened to a greater degree than those of longer wave-length. This agrees with a previous observation for the principal series of cæsium in the furnace, and indicates a shift of the maximum with change of temperature similar to that shown in the spectrum of an incandescent solid. The close resemblance in structure between the spectra of calcium, strontium, and barium permits a comparison of the homologous lines of the three elements. Such lines are found to be affected in the same way in the furnace spectra.

From the point of view of wave-length measurement, the elimination of dissymmetry in line structure and the resolution in the furnace of groups of lines which form diffuse blends in the arc in air are of especial interest in these spectra.

Elimination of Pole Effect from Secondary Standards of Wave-Length.[1]

By Harold D. Babcock and Charles E. St. John.

THE arc recommended by the International Committee on Wave-Lengths has been tested for pole effect. This disturbing influence is found to persist even at the center of the arc to the amount of 0.006 Å. The 6 mm., 6 ampere Pfund arc is also affected to about the same extent at its center, but by making it 12 mm. long and using 5 amperes or less, a central zone is found free from pole effect. That the values thus obtained are the fundamental wave lengths of the lines, is confirmed by observations upon arcs having negative poles of various constitution, operated so as to be free from pole effect. The importance of using fundamental wave-lengths as standards is pointed out.

An Optical Trough for High-School and College Demonstrations.[1]

By Ralph S. Minor.

THE optical trough is rectangular in form, about 3 inches wide and 3½ inches high and 26 inches long in size. The bottom is of metal and the sides and one end are plate glass. The other end carries a planomeniscus lens of less than 2 mm. thickness. The trough is suspended on a special tripod stand and the height may be regulated by means of an adjustable collar so that the trough may be rotated about a vertical axis after the height has been fixed.

[1] Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

This optical trough with its various accessories is designed to serve three purposes: First, as a piece of demonstration apparatus for a first course in physics, second, for the demonstration of a variety of phenomena which are beyond the scope of the first course; third, as a piece of laboratory apparatus for a course in college physics. The optical trough supplements the Hartl optical disk and has the advantage of using the entire wave front and not merely a section of it. The visibility of a beam of light passing through the trough is slightly greater than that produced by the Hartl disk with the same illumination.

The accessories include: plane and spherical mirrors, a plano-convex glass lens, double convex water lens, double concave air lens, trial case lenses and cylinders for studying the refraction of the eye, image and object screens for studying refraction at a single spherical surface, and models of both Huyghens and Ramsden eye pieces which may be used either in air or in water.

UNIVERSITY OF CALIFORNIA.

NOTE ON THE USE OF THE ELECTROMETER FOR MEASURING THE CONDUCTIVITY OF GASES.[1]

BY J. A. GILBREATH.

IN carrying out experiments on the conductivity of gases, a tube containing two parallel plate electrodes separated by a few millimeters is sometimes used. One of the plates is connected to an electrometer and the other to one terminal of a storage battery. H. S. Anderson used such a tube in working with the photoelectric effect and reports finding a current through the tube in a direction opposed to the E.M.F. of the battery.[2] This effect can be explained on the basis of induction. The two parallel plates constitute a condenser. If the E.M.F. of the battery decreases, the potential of the connected plate falls and this releases a part of the bound charge on the second plate. Thus, if one plate is connected to the positive pole of the battery it would have a positive charge and the second plate would have an equal negative charge. If the potential of the positive plate diminish it is unable to hold so large a bound charge on the second plate. Part of the negative charge from the second plate then flows into the electrometer giving the appearance of a current opposed to the E.M.F. of the battery.

If a battery is furnishing current, its E.M.F. is very apt to decrease. In the article referred to the cells were furnishing current through a water resistance which constituted a potentiometer system by which any desired E.M.F. could be applied to the tube. Under these conditions it is not strange to find a sensitive electrometer showing a negative " current " when a positive potential is applied.

[1] Abstract of a paper presented at the Stanford University meeting of the Physical Society, April 7, 1917.

[2] PHYS. REV., Vol. XXXV., October, 1912.

In using a tube of this kind with an electrometer whose sensibility was 4,000 divisions per volt the writer found that a potentiometer arrangement for varying the voltage was not satisfactory. The cells continually ran down, showing the negative effect. It was found more satisfactory to vary the voltage by varying the number of cells used. Before connecting the battery to the tube a small current was drawn through a high resistance for two or three minutes then the battery was allowed to recover for two or three minutes, and then connected to the tube. The negative effect then entirely disappeared and the electrometer showed no deflection.

PHYSICS LABORATORY,
UNIVERSITY OF WASHINGTON.

ERRATUM.

Vol. IX, April, 1917, abstract by I. G. Priest entitled "A Proposed Method for the Photometry of Lights of Different Colors, II.," p. 341, fourth line from bottom, V^{λ} should read V_{λ}.

On page 344,

"Table of Values of Ratio $\frac{\text{Candle power at 64.91 volts}}{\text{Candle power at 99.86 volts}}$"

should be heading for table on page 345.

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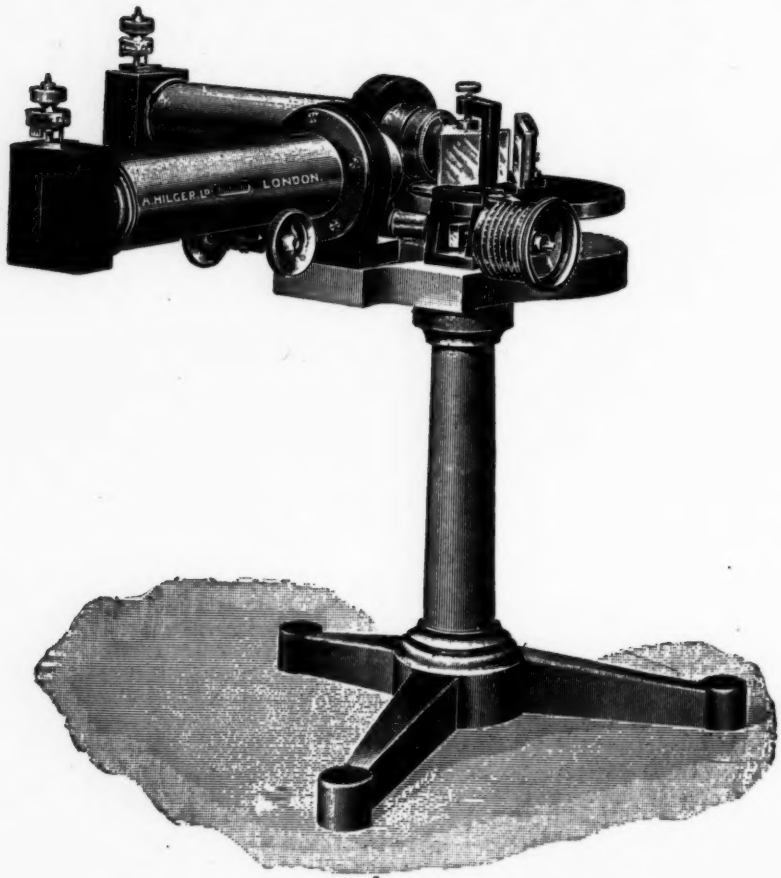

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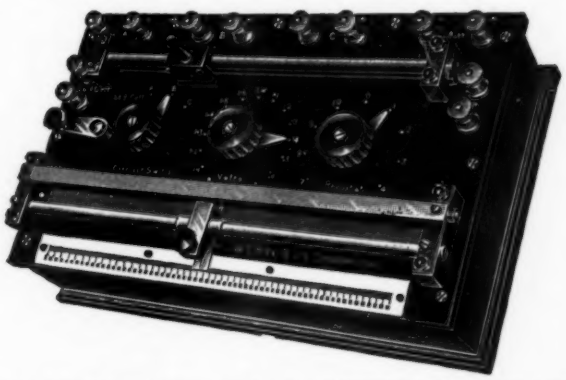

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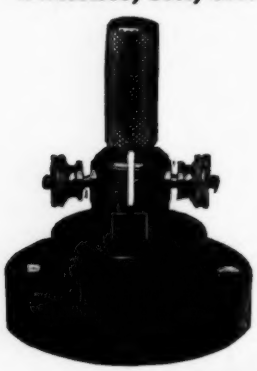

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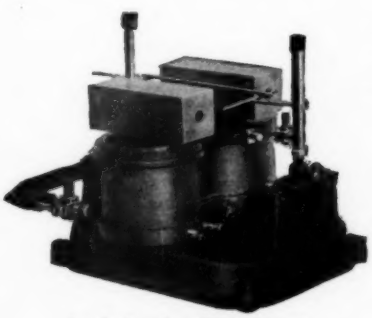

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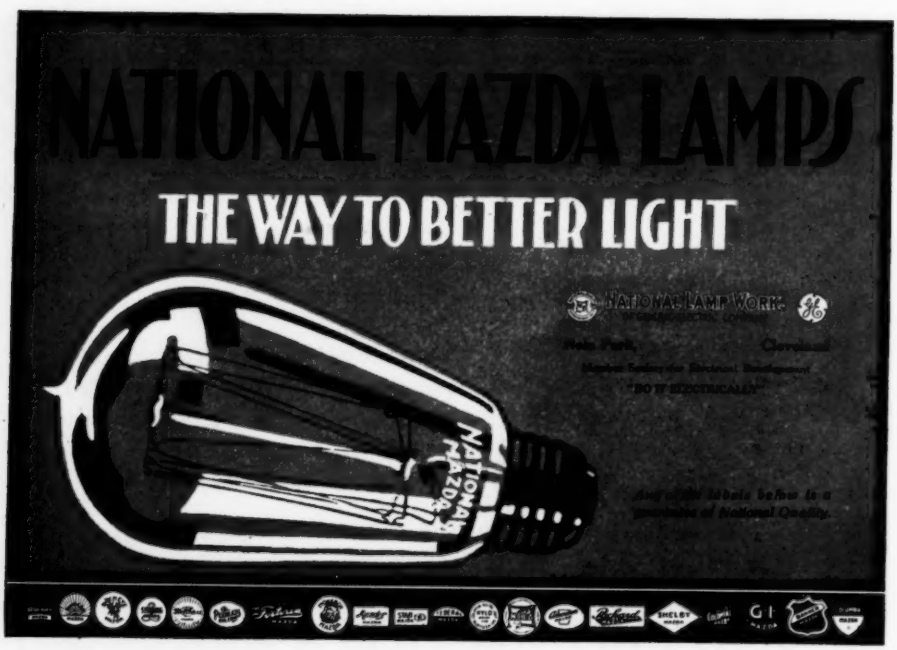

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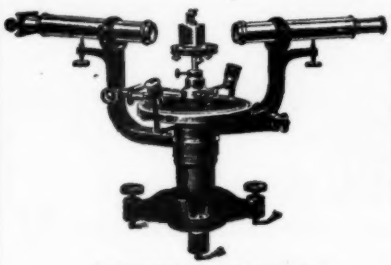

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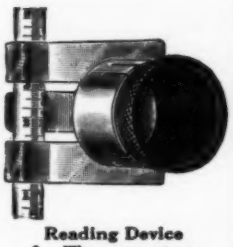

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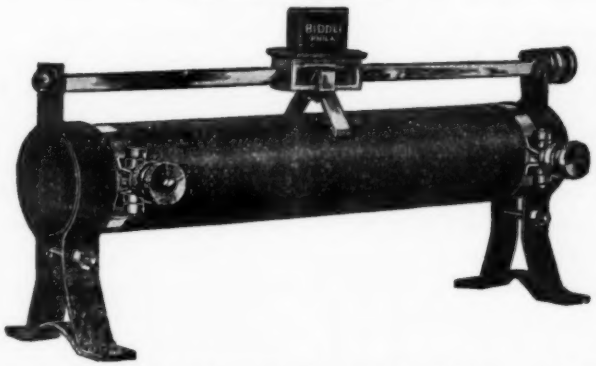

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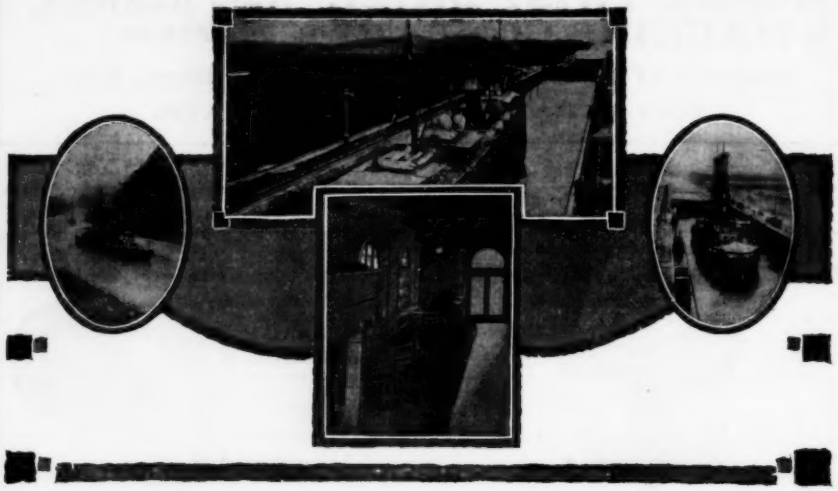

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